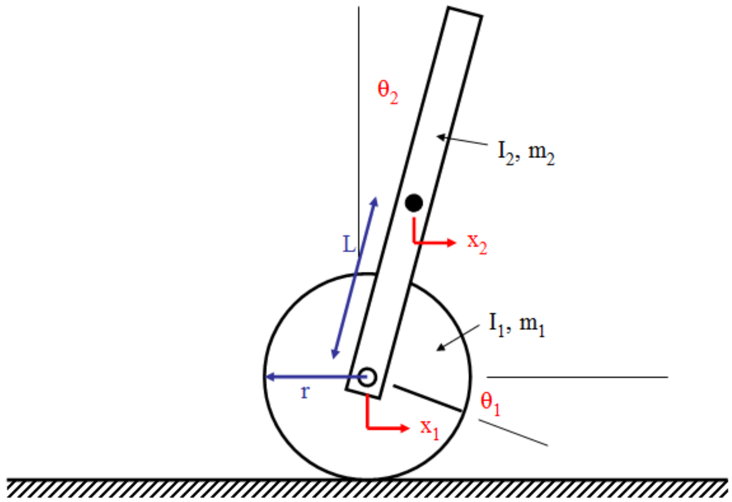
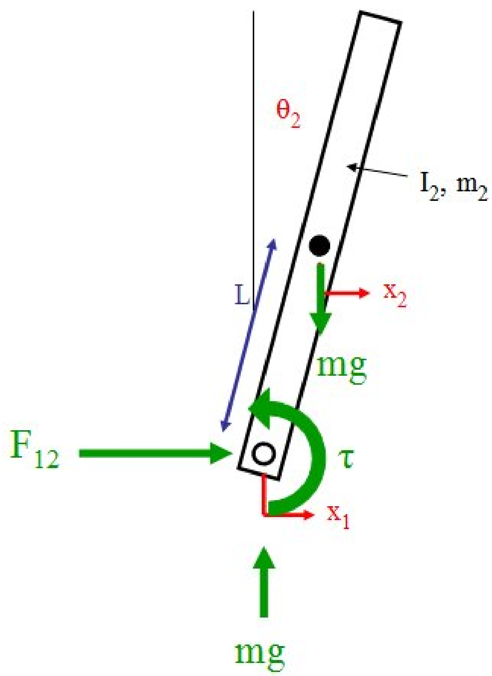
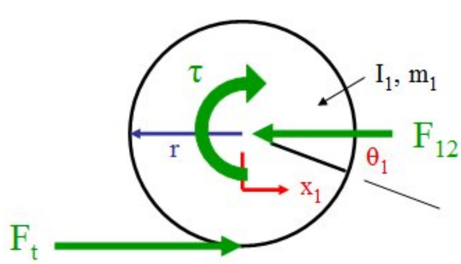


θ2
I2, m2
L
x2
I1, m1
r
θ1
x1



θ2
I2, m2
L
x2
mg
F12
τ
x1
mg



τ
I1, m1
r
F12
θ1
x1
Ft

Kinematic Relationships (no slip):

$$x_1 = r\theta_1$$
$$x_2 = x_1 + l\theta_2 = r\theta_1 + l\theta_2$$

Stick Linearized Dynamics:

$$m_2\ddot{x}_2 = F_{12}$$
$$I_2\ddot{\theta}_2 = m_2 gl\theta - F_{12}l - \tau$$

Wheel Linearized Dynamics:

$$m_1\ddot{x}_1 = F_t - F_{12}$$
$$I_1\ddot{\theta}_1 = -F_t r + \tau$$

…some algebra omitted…

Combined Equations of Motion:

$$H_1\ddot{\theta}_1 + H_3\ddot{\theta}_2 = \tau$$
$$H_3\ddot{\theta}_1 + H_2\ddot{\theta}_2 - a\theta_2 = -\tau$$

$$H_1 = I_1 + m_1 r^2 + m_2 r^2$$
$$H_2 = I_2 + m_2 l^2$$
$$H_3 = m_2 rl$$
$$a = m_2 gl$$

A transfer function from θ_2 to θ_1 can be found by adding the two combined equations of motion:

$$(H_1 + H_3)s^2\theta_1 + \left[(H_2 + H_3)s^2 - a\right]\theta_2 = 0$$
$$\Rightarrow \frac{\theta_1}{\theta_2} = -\frac{(H_2 + H_3)s^2 - a}{(H_1 + H_3)s^2}$$

This can now be subbed in to the first equation of motion:

$$H_1 s^2\theta_2\frac{\theta_1}{\theta_2} + H_3 s^2\theta_2 = \tau$$

$$\left(-\frac{H_1 s^2\left[(H_2+H_3)s^2-a\right]}{(H_1+H_3)s^2}+H_3 s^2\right)\theta_2=\tau$$

$$\left(-\frac{(H_1H_2+H_1H_3)s^2-H_1a}{(H_1+H_3)}+\frac{(H_1H_3+H_3^2)s^2}{(H_1+H_3)}\right)\theta_2=\tau$$

$$\left(\frac{(H_3^2-H_1H_2)s^2+H_1a}{(H_1+H_3)}\right)\theta_2=\tau$$

Lastly, pop a negative sign out of everything on the left-hand side:

$$-\left(\frac{(H_1H_2-H_3^2)s^2-H_1a}{(H_1+H_3)}\right)\theta_2=\tau$$

The transfer function from τ to θ_2 is thus:

$$\frac{\theta_2}{\tau}=-\frac{(H_1+H_3)}{(H_1H_2-H_3^2)s^2-H_1a}$$

Since we know that $(H_1H_2 - H_3^2)$ is a positive quantity (given in the problem statement, or derivable from the mass and inertia values), this is a second order system with two real poles, one of which is in the right half plane. This gives rise to the open-loop instability of the balancing platform.