A Modified Sychronous Current Regulator for Brushless Motor Control

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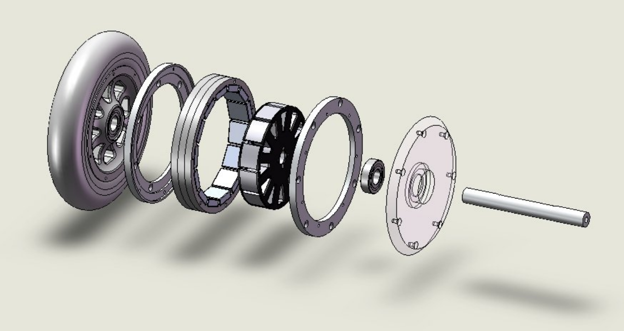
Overview

- This work details a torque controller for brushless Permanent Magnet Synchronous Motors (PMSM).

- Methods of controlling PMSM:
  - Brushless DC Control
  - Field-Oriented Control (FOC): Synchronous Current Regulator (SCR)

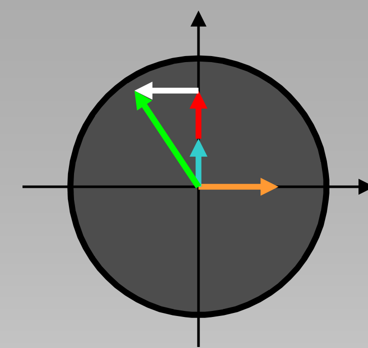
- The author's contribution is a modified SCR that:
  - uses Hall effect sensors (instead of an encoder).
  - is more computational efficient (low-cost processing).
  - has the potential for improved transient response.

- The design of the controller and an experimental application to low-cost personal transportation will be detailed.

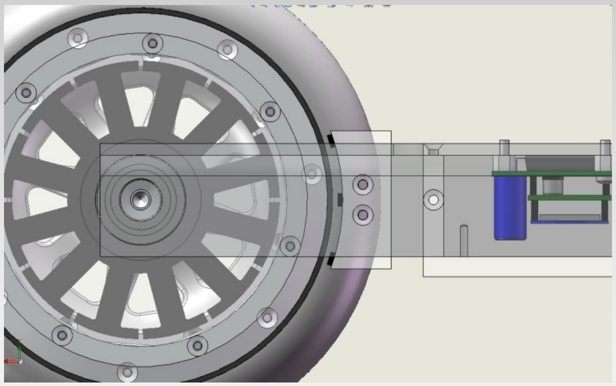


Outline

Theoretical Analysis
- Permanent Manget Synchronous Motor Model
- Field Oriented Control Principles
- Synchronous Current Regulator (SCR)
- Modified Synchronous Current Regulator (mSCR)



Applied Analysis
- Plant Information
- Controller Hardware
- Controller Design
- Controller Simulations: SCR and mSCR
- Experimental Testing and Data

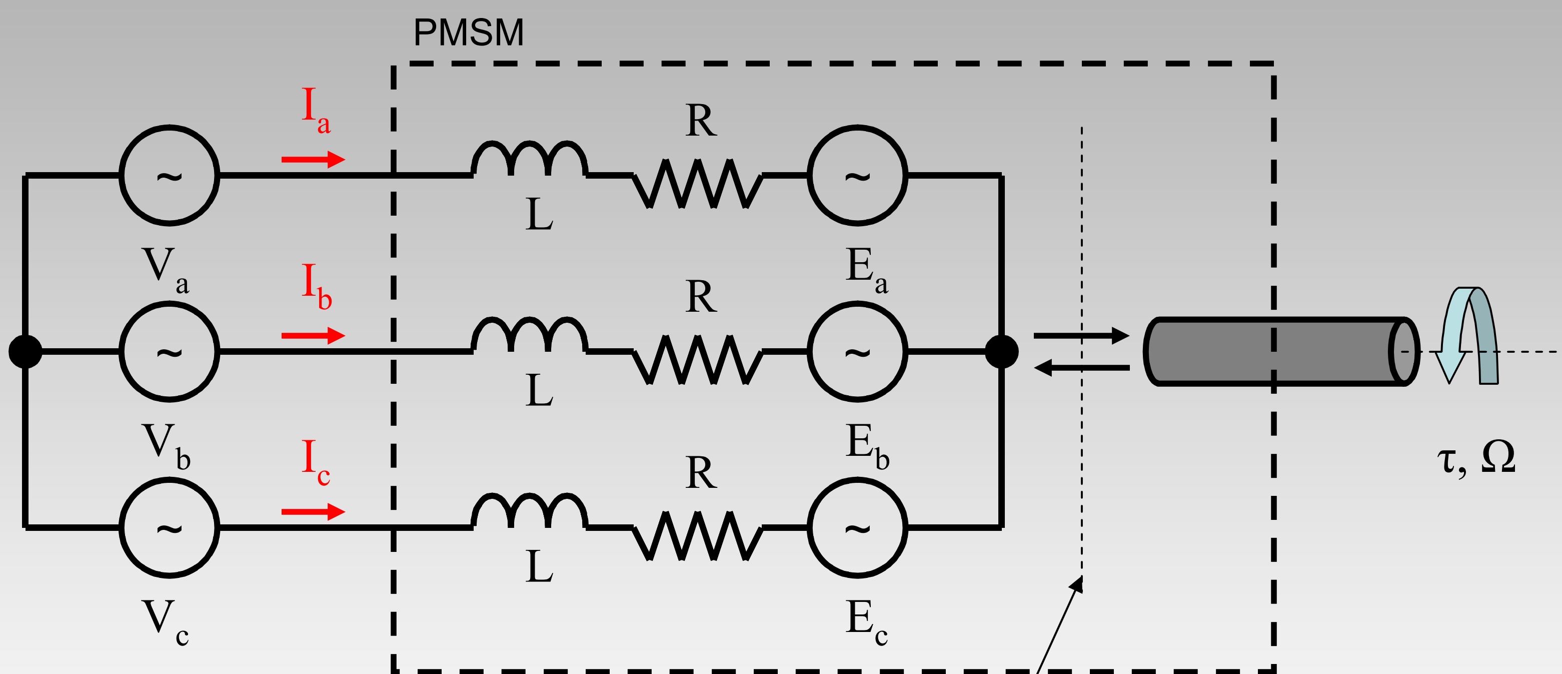


- Future Work
- Questions / Feedback

- Motor Control Overview
- Current Sensing
- Simplified Plant Closed-Loop Transfer Function and Root-Locus
- A more fair transient response comparison.
- High-Speed Operation
- Error Handling and Failsafes
- Connection to Adaptive Feed-Forward Cancellation (AFC)

PMSM Model

Three-phase permanent magnet synchronous motor (PMSM) electromechanical model:



Power Conversion: $$\tau = \frac{\vec{I_a} \cdot \vec{E_a} + \vec{I_b} \cdot \vec{E_b} + \vec{I_c} \cdot \vec{E_c}}{\Omega}$$

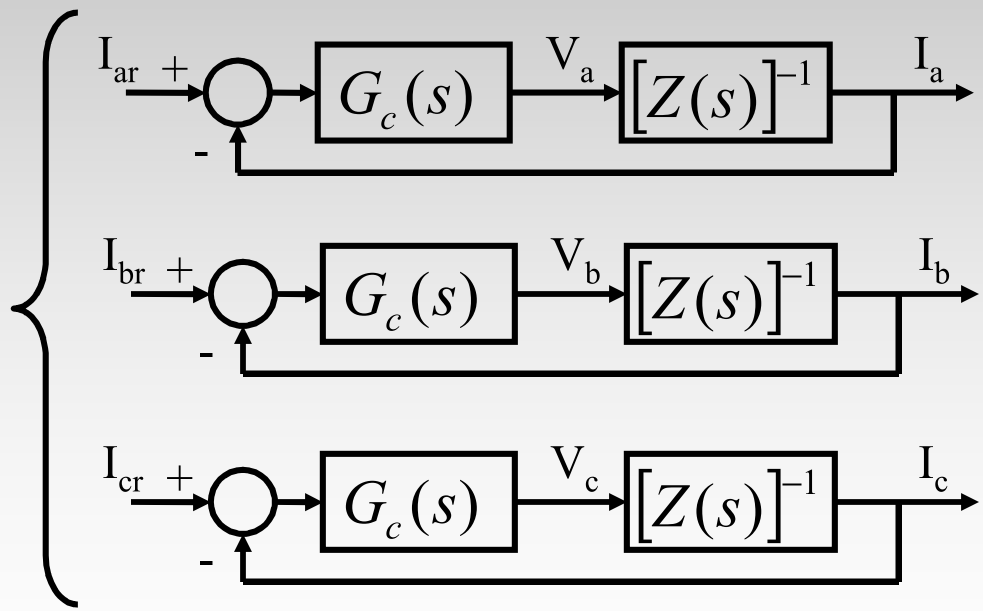
PMSM Model

- To control torque, both the phase and the magnitude of current must be controlled.

- One option: high-bandwidth current controllers on each phase of the brushless motor. The closed-loop bandwidth must be significantly faster than the commutation of the motor (the AC frequency):

AC References:

$$I_{xr} = |I|\sin(\omega t + \phi_x),$$
$$x = \{a, b, c\}$$



Field-Oriented Control Principles

By exploiting symmetry of the three-phase variables and transforming to the reference frame of the rotor, the controller can act on quantities which are DC in steady-state operation.

(Similar to adaptive feed-forward cancellation with sinusoidal input.)

⇩

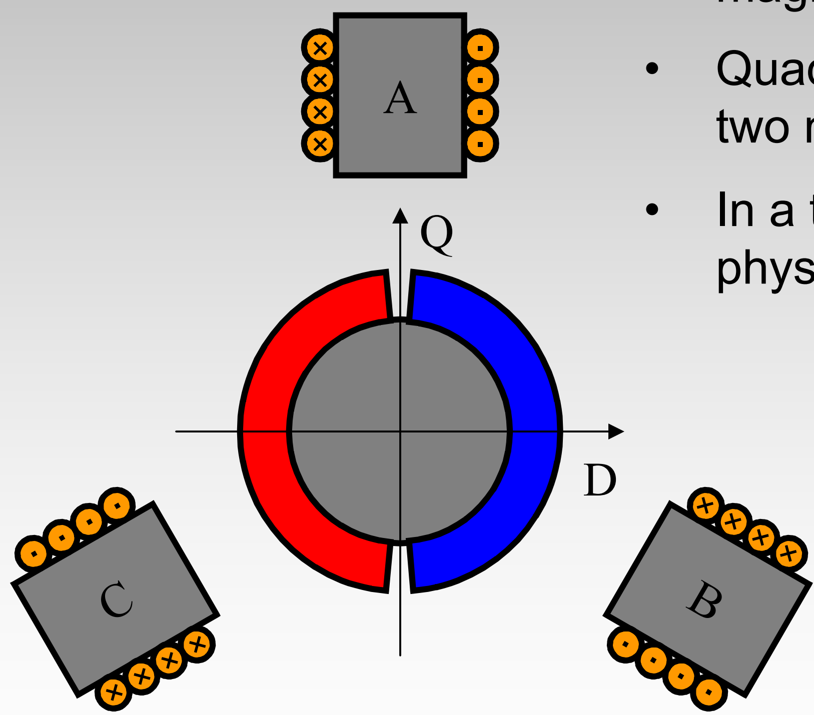
Field-Oriented Current control works without the need for high-bandwidth control loops.

- Easier to implement on fixed-point, low-cost microcontrollers.
- Better high-speed performance.

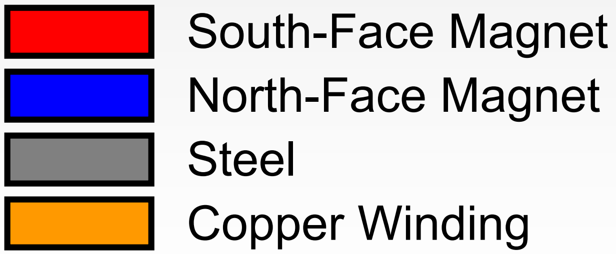
Field-Oriented Control Principles

Vector Motor Quantities, D/Q Axes

- Controller operates in a two-dimensional coordinate system that is attached to the rotor: rotor/synchronous reference frame.



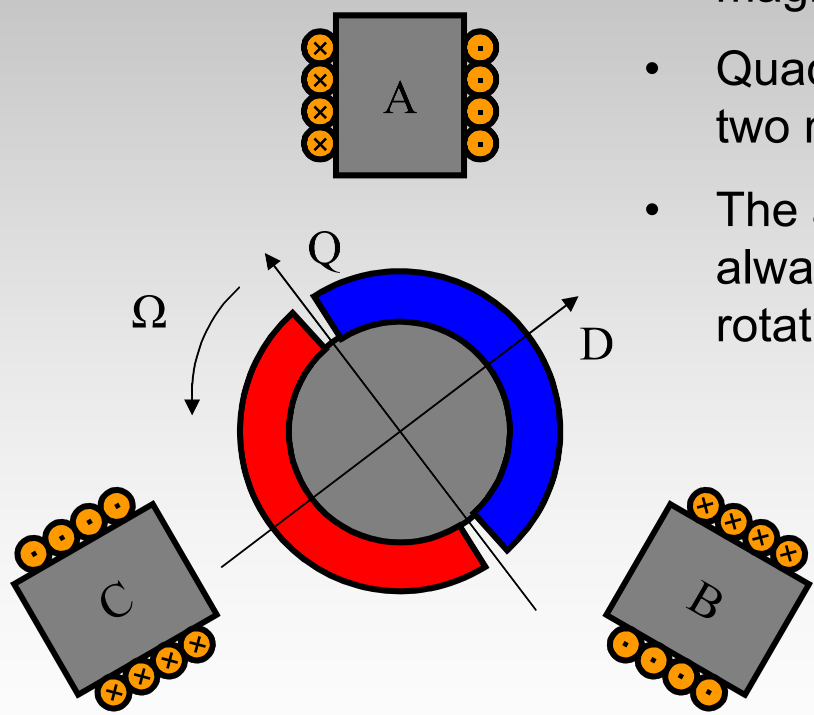
- Direct (D) Axis: Aligned with a North magnet pole.
- Quadrature (Q) Axis: Exactly between two magnet poles.
- In a two-pole motor, they are physically perpendicular.



Field-Oriented Control Principles

Vector Motor Quantities, D/Q Axes

- Controller operates in a two-dimensional coordinate system that is attached to the rotor: <u>rotor reference frame</u>.

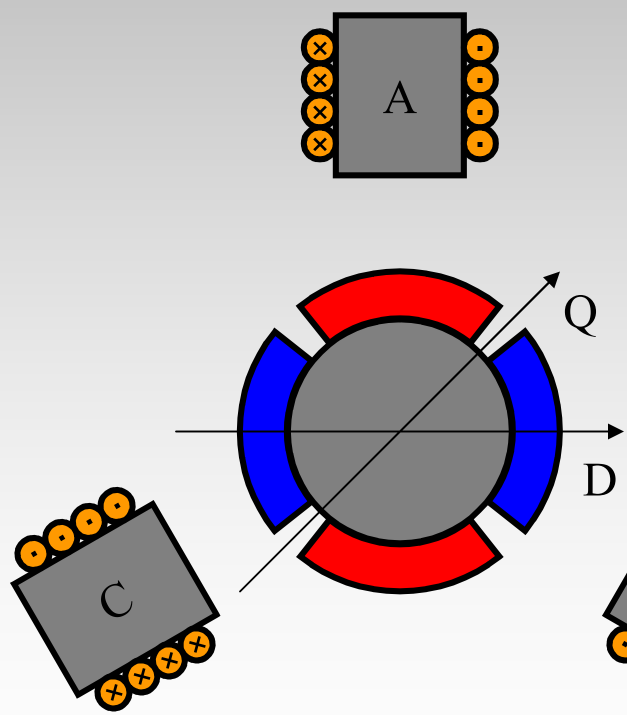


- Direct (D) Axis: Aligned with a North magnet pole.
- Quadrature (Q) Axis: Exactly between two magnet poles.
- The axes are <u>attached to the rotor</u>. Q always leads D in the direction of rotation.

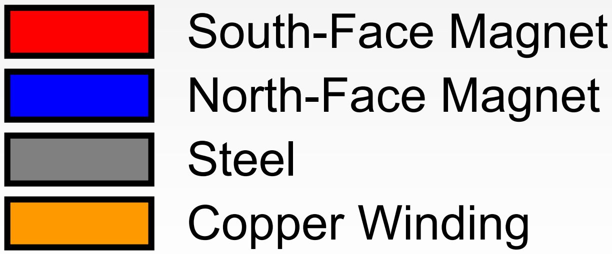
Field-Oriented Control Principles

Vector Motor Quantities, D/Q Axes

- Controller operates in a two-dimensional coordinate system that is attached to the rotor: rotor reference frame.



- Direct (D) Axis: Aligned with a North magnet pole.
- Quadrature (Q) Axis: Exactly between two magnet poles.
- In a four-pole motor, they are separated by 45º mechanical. They are always separated by 90º electrical.



Field-Oriented Control Principles

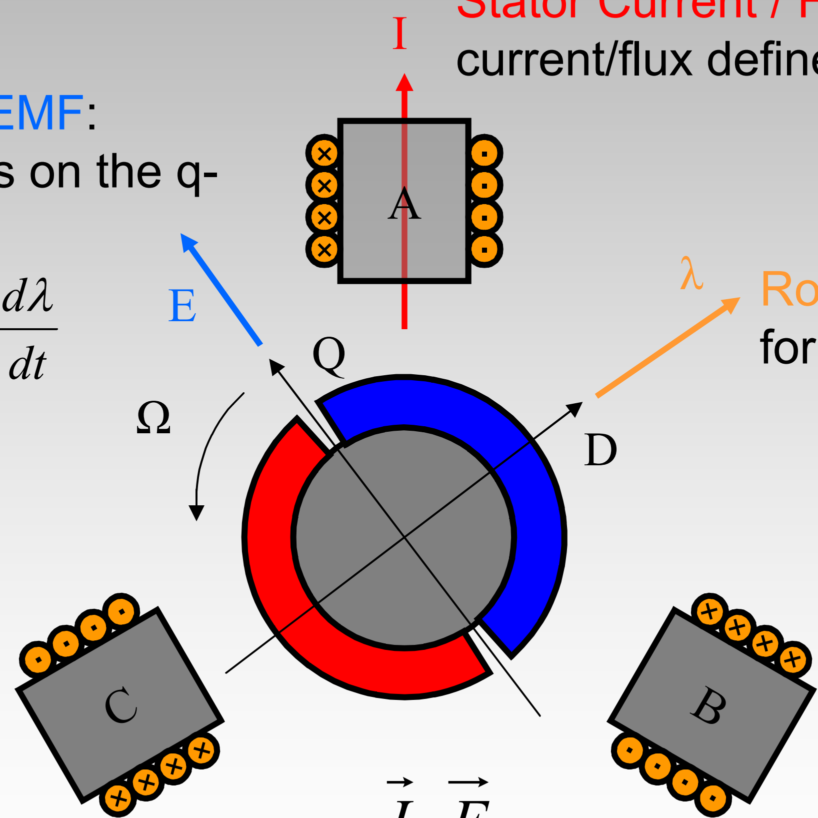
Vector Motor Quantities, D/Q Axes

- All motor quantities that have “direction” can be projected onto the d/q axes as vectors:

Stator Current / Flux: Vector sum of coil current/flux defined by right hand rule.

Back EMF: Always on the q-axis.

$$E = \frac{d\lambda}{dt}$$



Rotor Flux Linkage: Always on the d-axis for a permanent magnet motor.

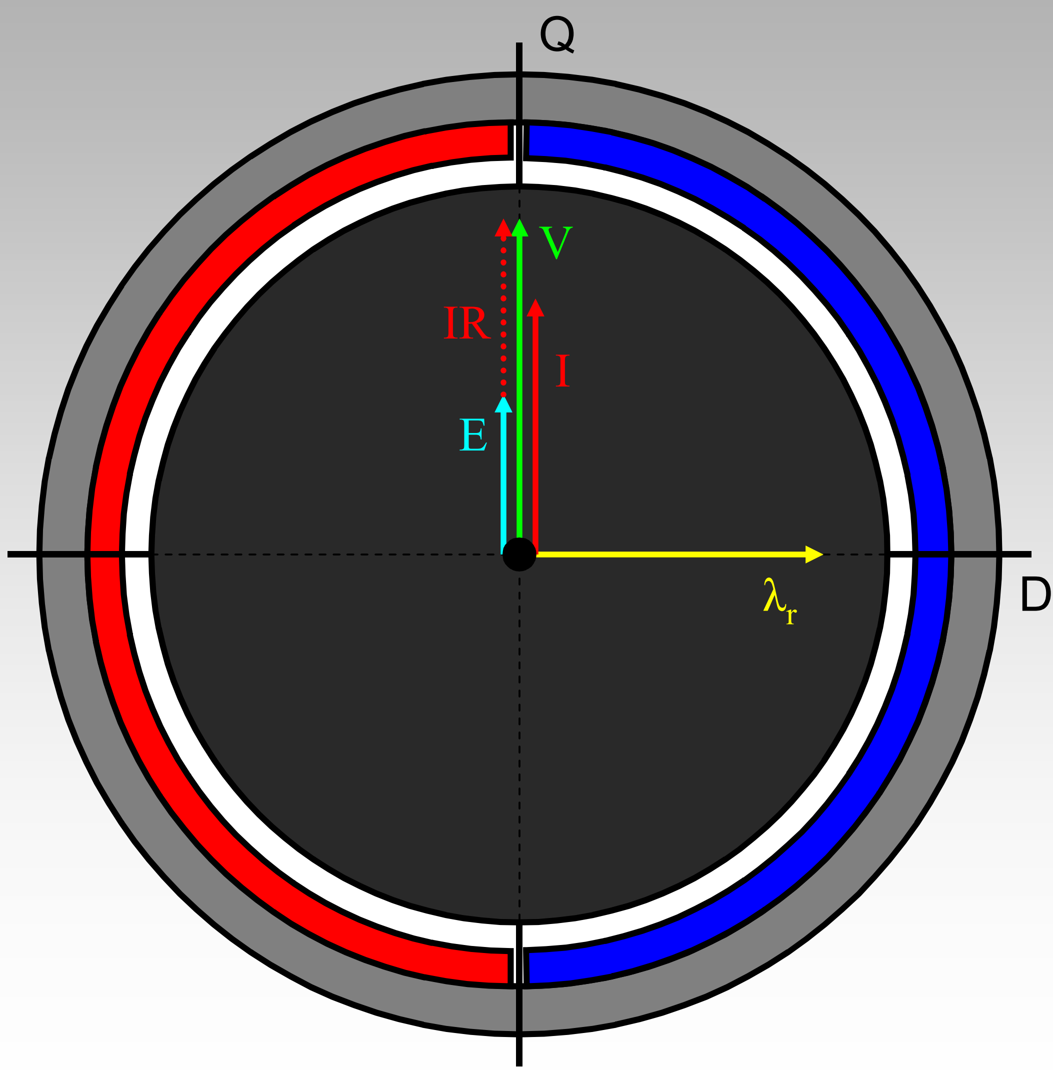
$$\lambda = N\phi$$

$$\tau = \frac{\vec{I} \cdot \vec{E}}{\Omega}$$

South-Face Magnet
North-Face Magnet
Steel
Copper Winding

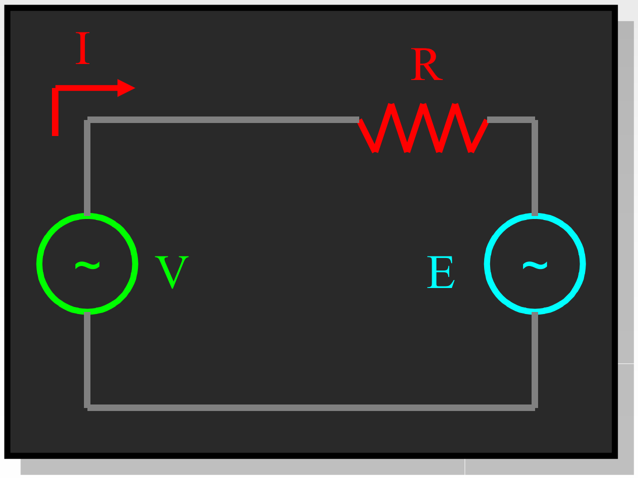
Field-Oriented Control Principles

Unrealistic Zero-Inductance Motor



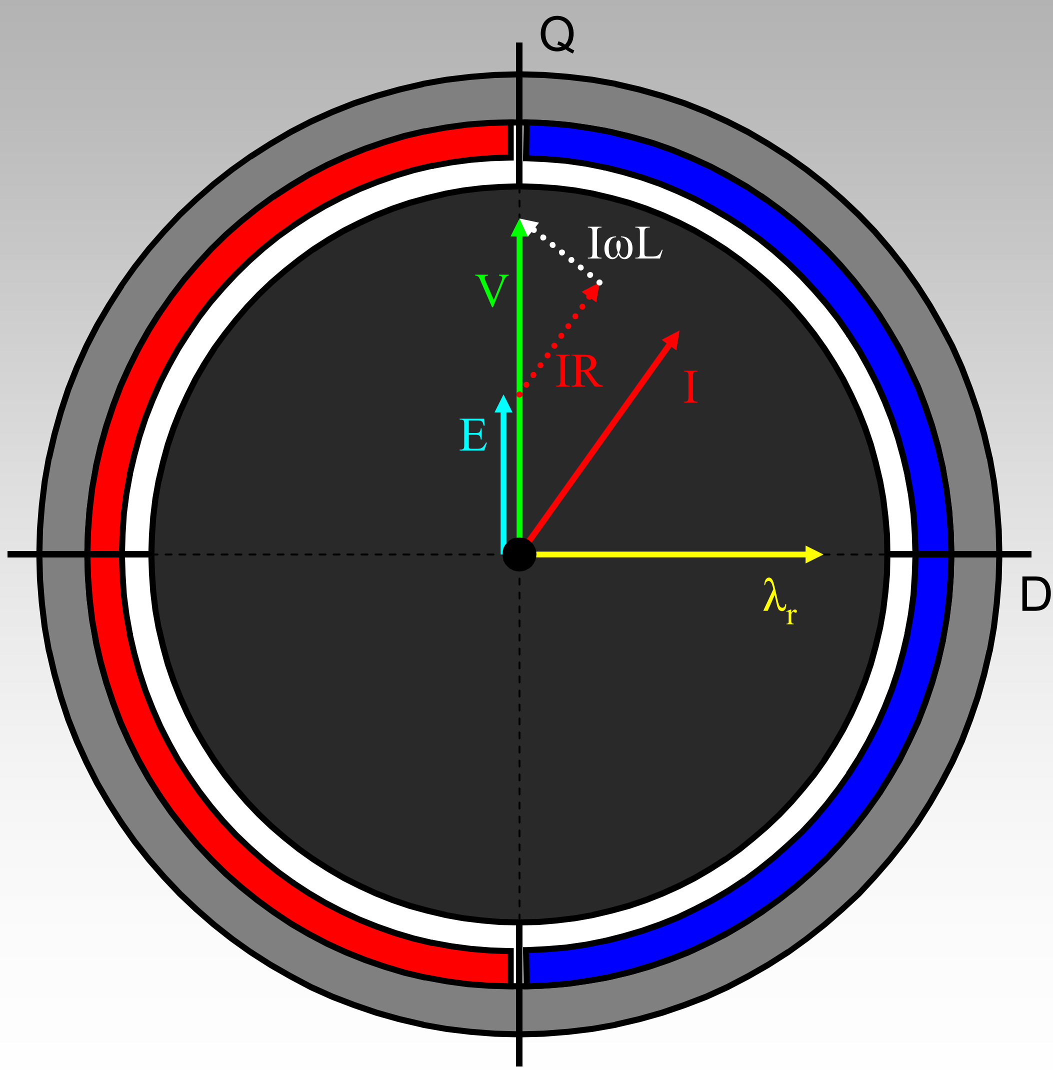
- Voltage applied in-phase with back-EMF.
- Current also in-phase with back-EMF.
- Torque per amp is optimal.
- Reasonable approximation if inductance or speed is low:

$$\omega L << R$$



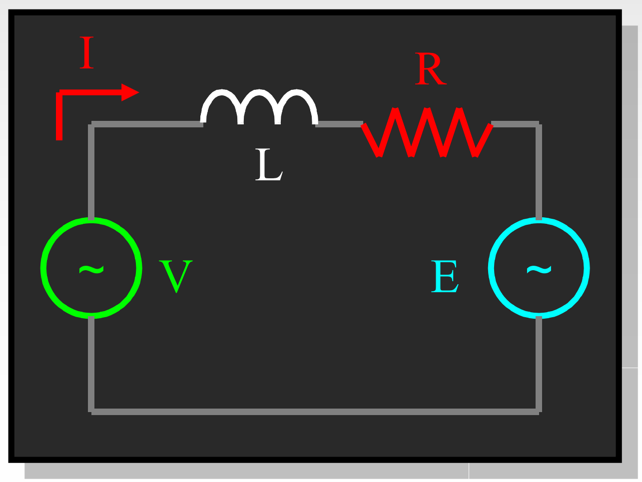
Field-Oriented Control Principles

Motor with Inductance



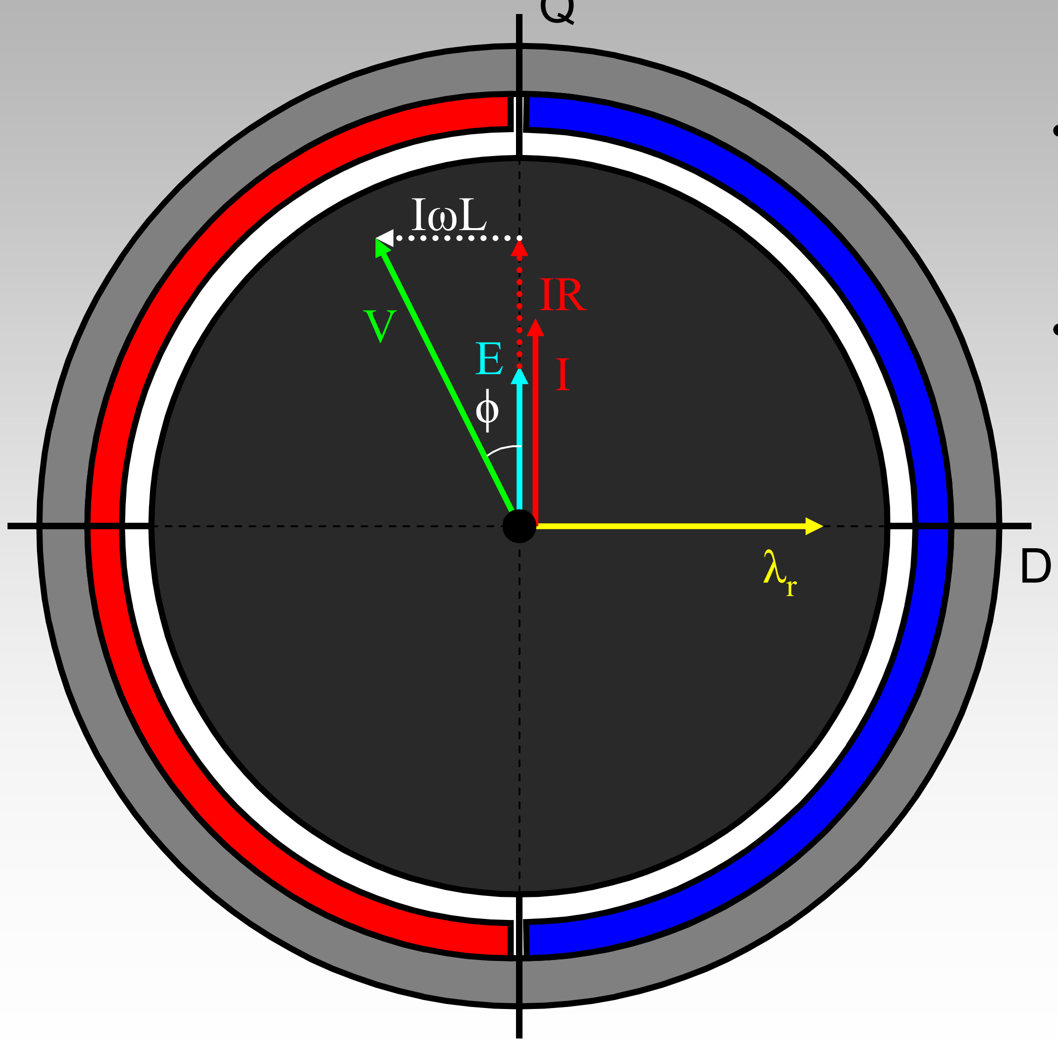
- Voltage applied in-phase with back-EMF.
- Current lags due to the motor inductance.
- Torque per amp is no longer optimal. Current and back EMF are not in phase:

$$\vec{I} \times \vec{E} \neq 0$$



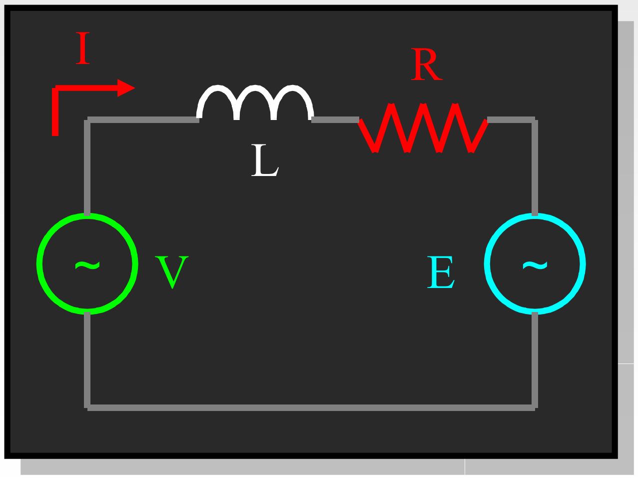
Field-Oriented Control Principles

Phase Advance to Correct for Inductance Lag



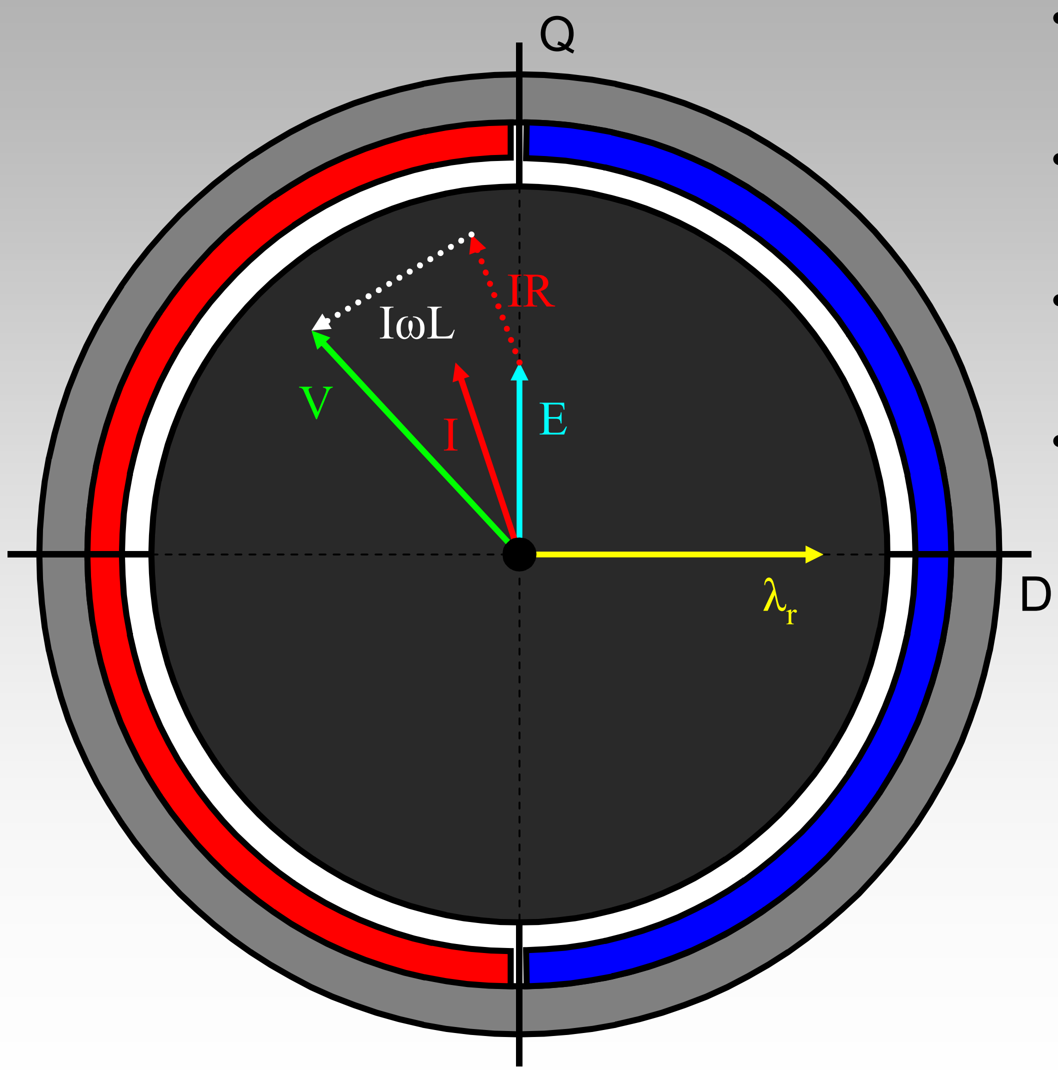
- Voltage applied ahead of back EMF.
- Current lags due to the motor inductance such that it is in phase with back EMF.
- Torque per amp is optimal.

$$\phi = f(V, I, \Omega, K_t, R, L, \ldots)$$

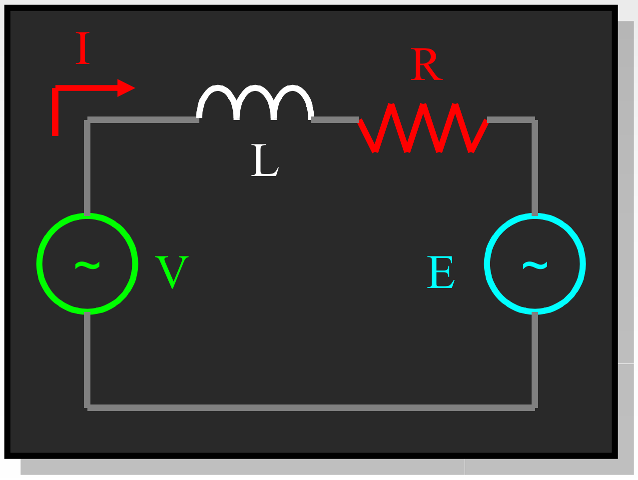


Field-Oriented Control Principles

Field Weakening for High-Speed Operation



- Voltage and current both lead back EMF.
- Stator flux counteracts rotor flux: “field weakening”
- Torque per amp is not optimal but…
- Maximum achievable speed per volt is higher.



Field-Oriented Control Principles

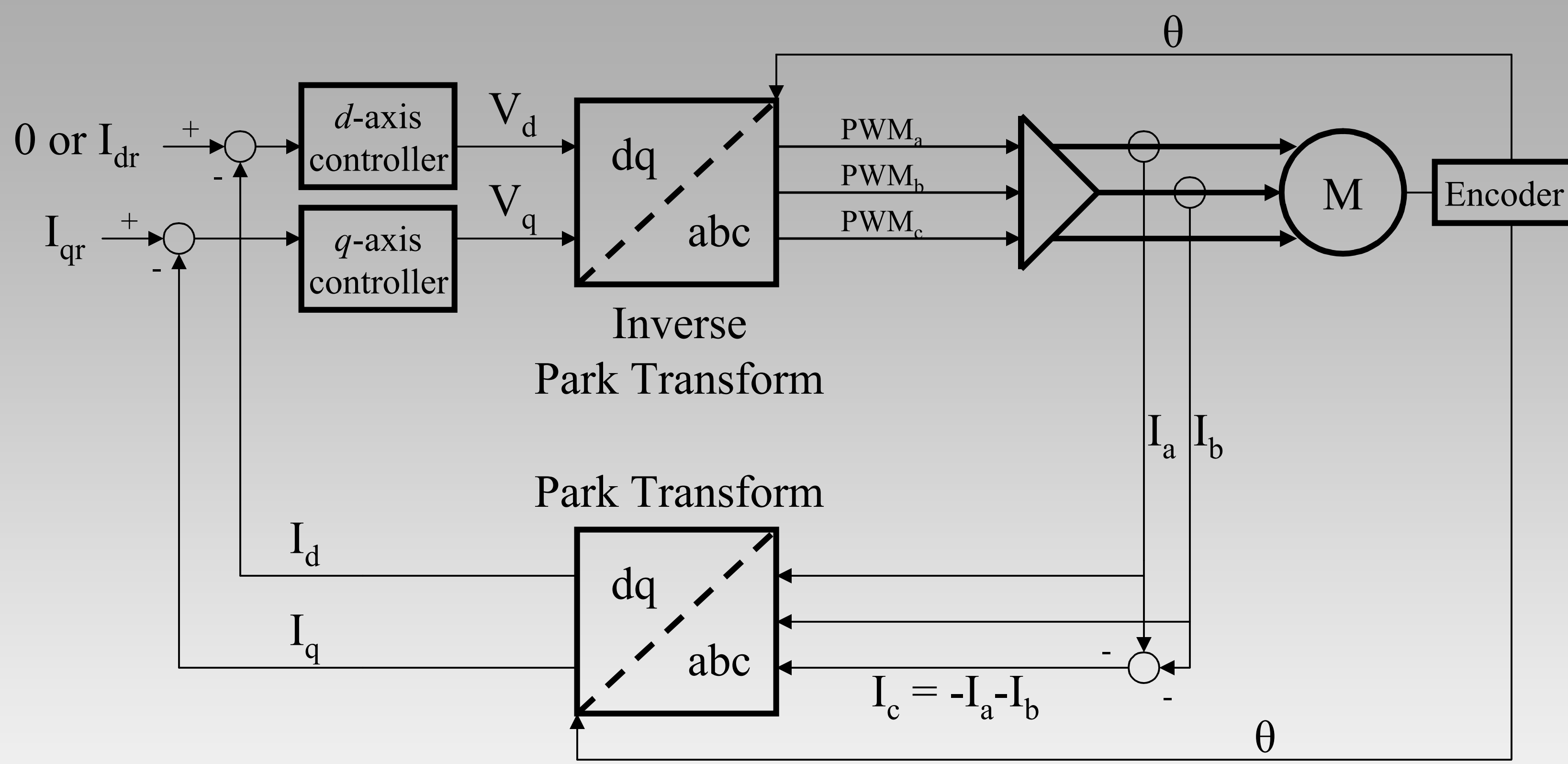
Park Transform / Inverse Park Transform

- Tranforms used to convert from/to stator frame {a,b,c} quantities to/from rotor frame {d,q} quantities.
- Require rotor position, θ, as an input.

$$\begin{bmatrix} x_d \\ x_q \\ x_0 \end{bmatrix} = T \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} \qquad T = \frac{2}{3}\begin{bmatrix} \cos\theta & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\ -\sin\theta & -\sin\left(\theta - \frac{2\pi}{3}\right) & -\sin\left(\theta + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

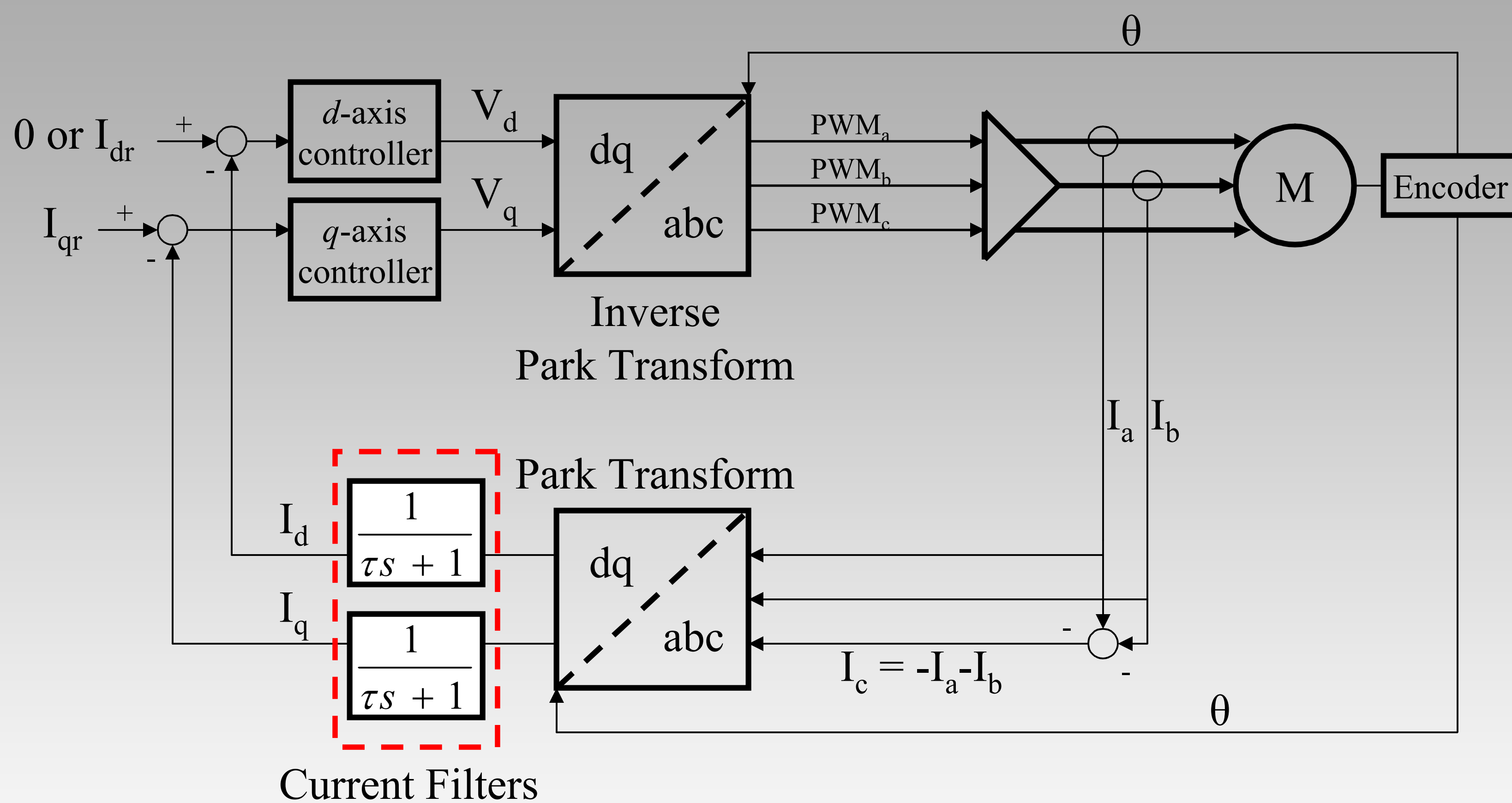
$$\begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} = T^{-1} \begin{bmatrix} x_d \\ x_q \\ x_0 \end{bmatrix} \qquad T^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta & 1 \\ \cos\left(\theta - \frac{2\pi}{3}\right) & -\sin\left(\theta - \frac{2\pi}{3}\right) & 1 \\ \cos\left(\theta + \frac{2\pi}{3}\right) & -\sin\left(\theta + \frac{2\pi}{3}\right) & 1 \end{bmatrix}$$

Synchronous Current Regulator



- Park and inverse Park transform convert into and out of rotor reference frame.
- Two “independent” controllers for the d- and q-axis.
- Requires rotor position, typically from an encoder or resolver.

Synchronous Current Regulator

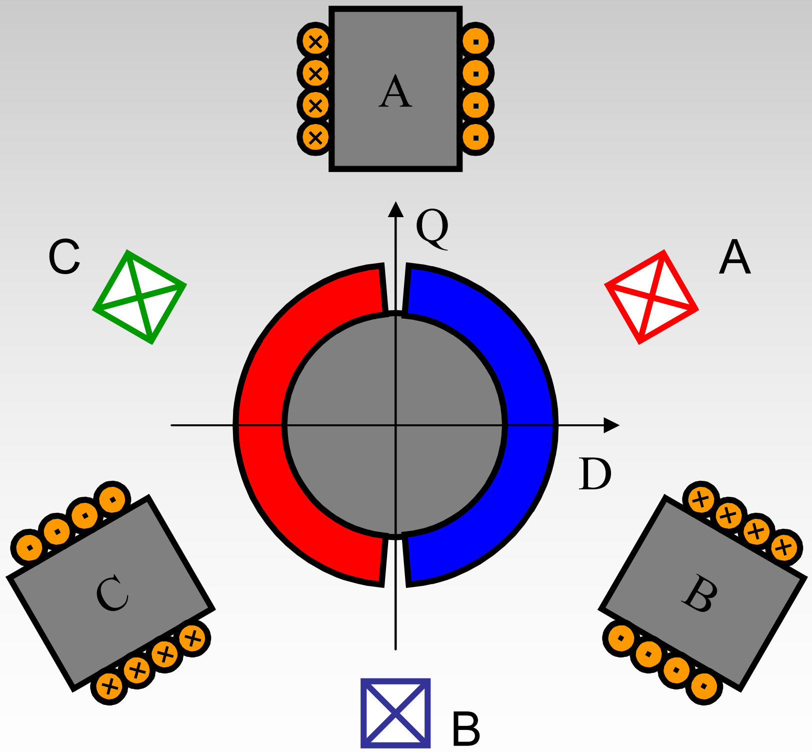


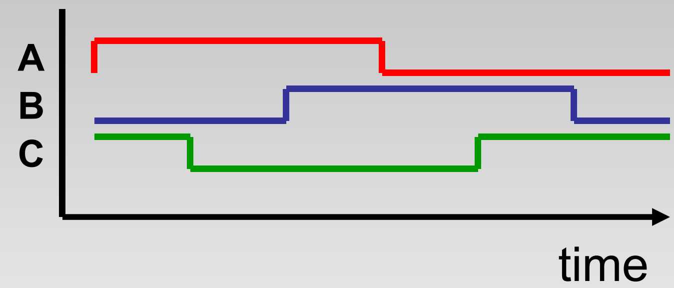
- Because the controllers run in the rotor frame, where values are “DC” in steady state, the controllers may operate at low bandwidth, below commutation frequency, and long time-constant current filtering can be implemented.

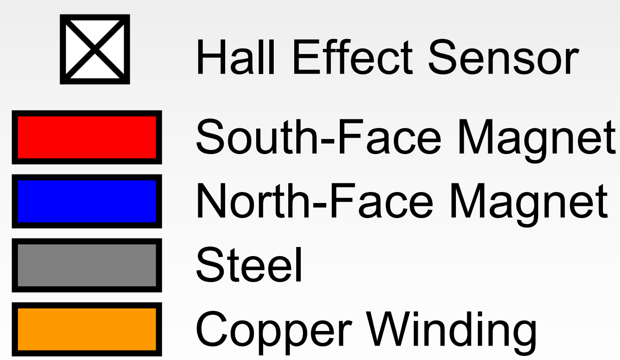
Modified Synchronous Current Regulator

Initial Motivation

- For sufficient resolution of rotor position, an encoder or resolver is typically required for field oriented control. (Sensorless techniques also exist.)
- However, less expensive motors use three Hall effect sensors to derive rotor position with 60º electrical resolution:



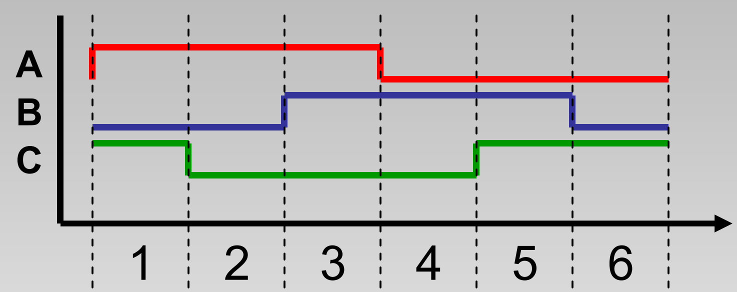




Modified Synchronous Current Regulator

Initial Motivation

In sensored brushless DC control, the six Hall effect sensor states directly map to phase voltage outputs.

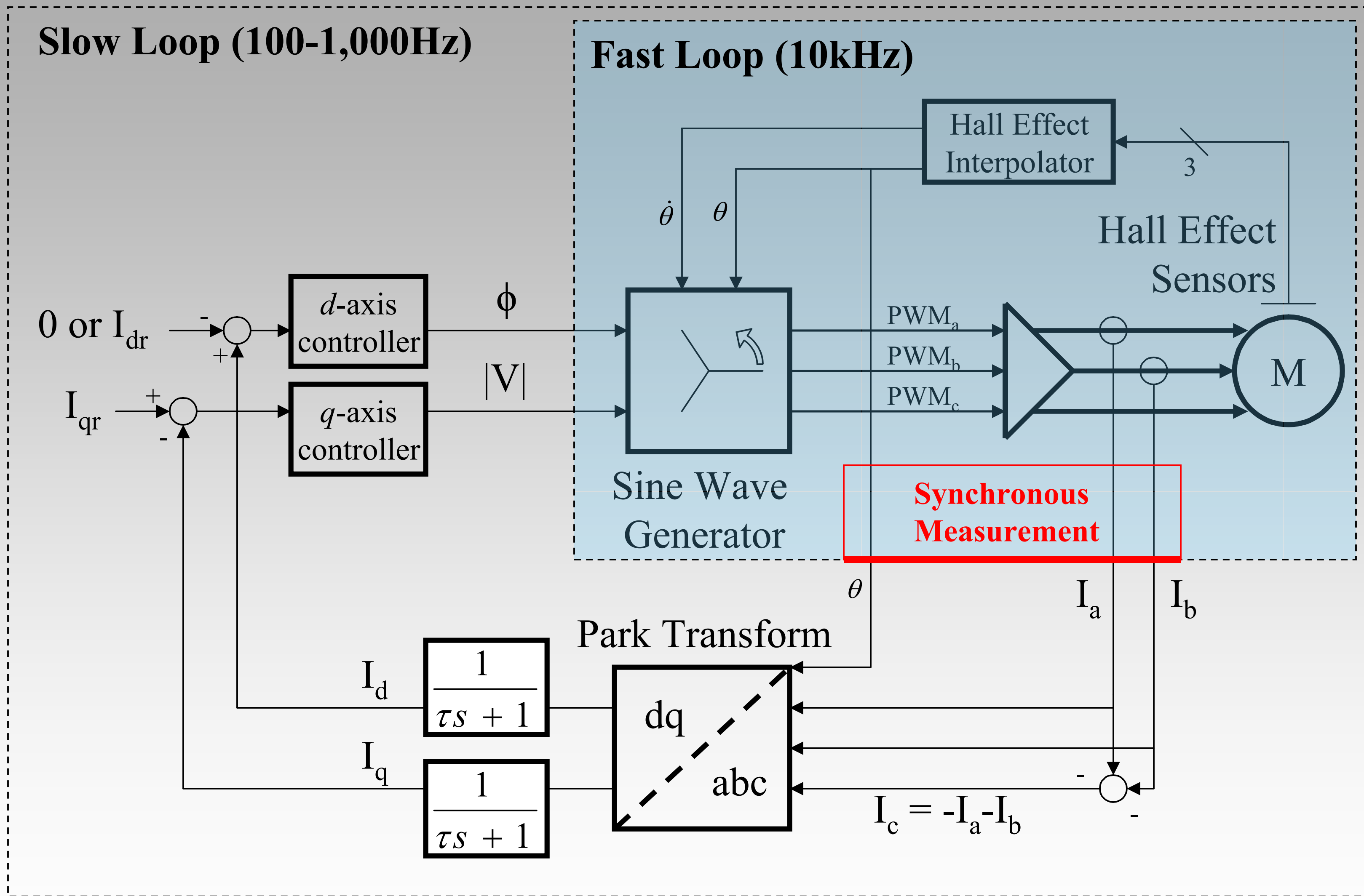


State	V_a	V_b	V_c
1	PWM	0V	High-Z
2	High-Z	0V	PWM
3	0V	High-Z	PWM
4	0V	PWM	High-Z
5	High-Z	PWM	0V
6	PWM	High-Z	0V

- Pros: very simple algorithm (state table), can run on low-cost processor.
- Cons: fixed timing, torque ripple, audible noise

Initial Motivation: Can the Synchronous Current Regulator be modified to work with Hall effect sensor inputs, with interpolation?

Modified Synchronous Current Regulator



Modified Synchronous Current Regulator

There are several *practical* differences:

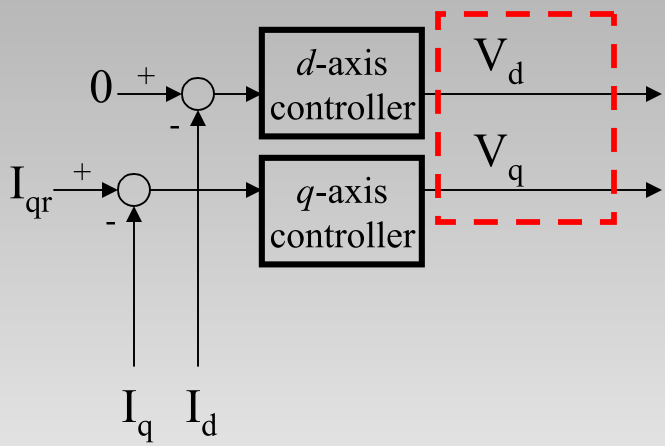
- The controller is explicity split into fast and slow loops; only PWM generation and rotor position estimatation need be in the fast loop.
- PWM generation is done by a sine table look-up, which is faster to compute than an inverse Park transform.
- The rotor position is estimated by interpolating between Hall effect sensor absolute states using the last known speed.
- As long as rotor position and phase currents are sampled synchronously by the slow loop, the slow loop bandwidth can be arbitrarily low.

$$\Downarrow$$

- The modified synchronous current regulator can be run on <u>fixed-point processors</u> to control <u>low-cost motors</u> with Hall effect sensors.
- It can achieve AC servo motor-like control with brushless DC motors.

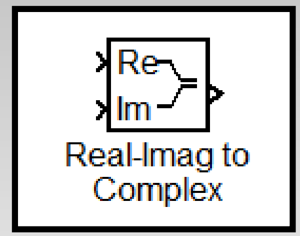
Modified Synchronous Current Regulator

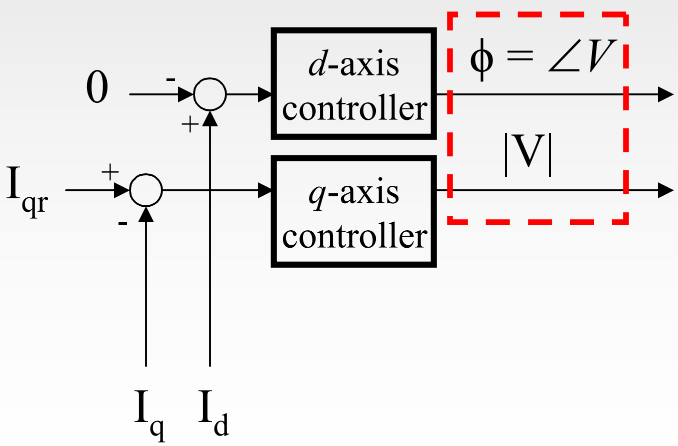
The primary *theoretical* difference is the controller outputs:



Standard SCR

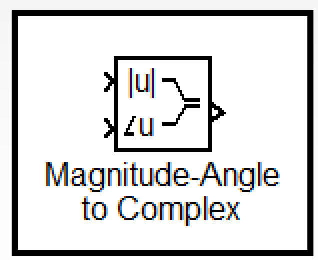
- V_d and V_q fully-define a voltage vector.
- D-axis gain: [V/A]
- Q-axis gain: [V/A]
- Simulate with:





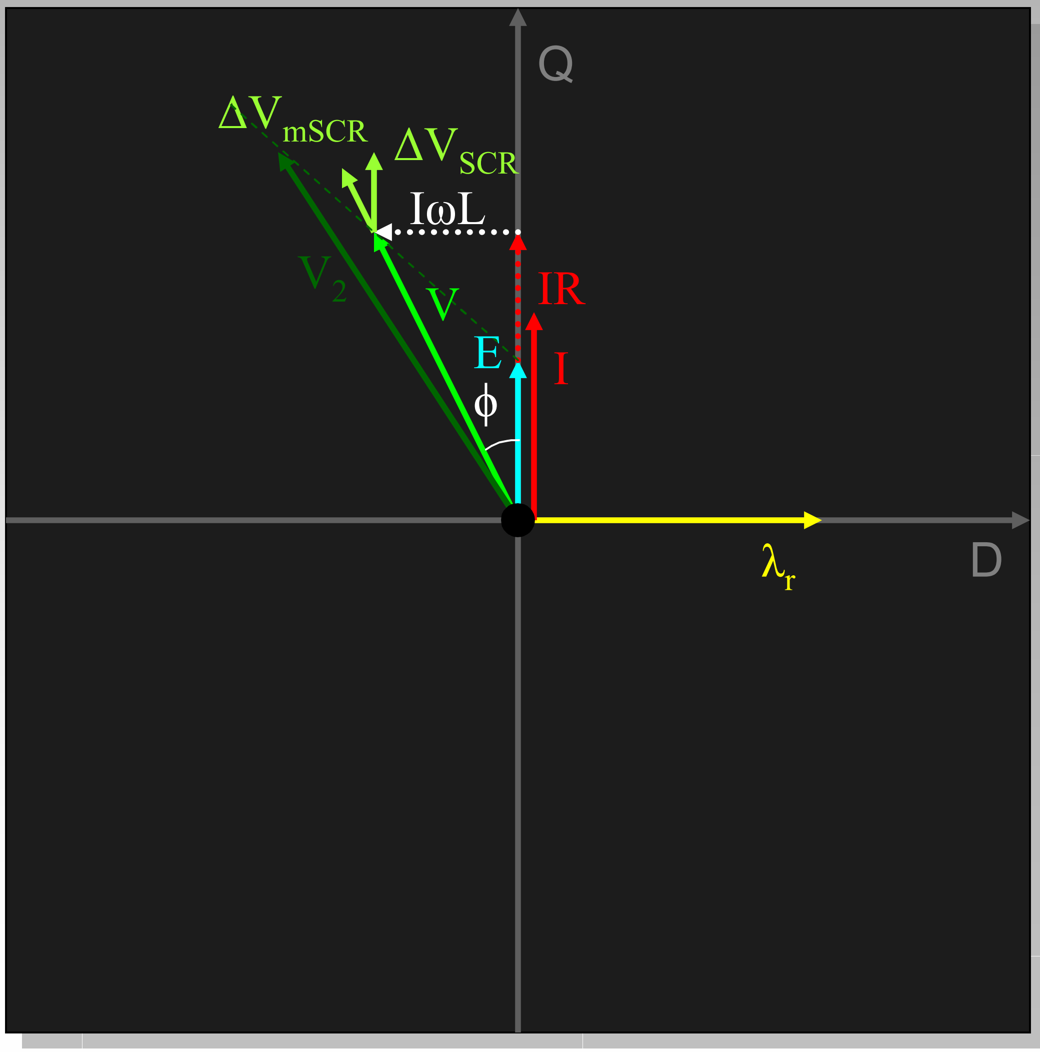
Modified SCR

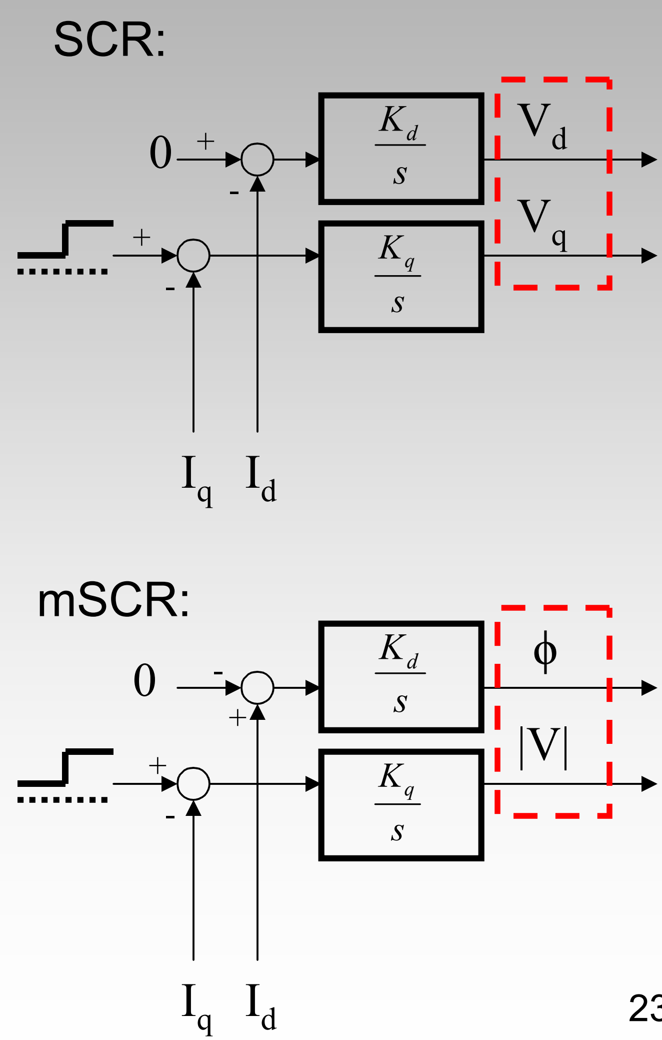
- $|V|$ and $\angle V$ fully-define a voltage vector.
- D-axis gain: [rad/A]
- Q-axis gain: [V/A]
- Simulate with:



Modified Synchronous Current Regulator

Consider a step increase in torque command via I_{qr}:





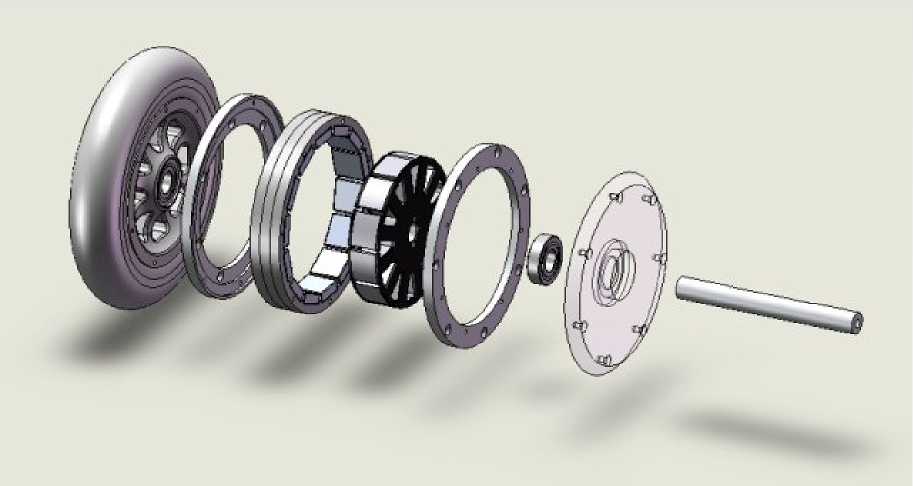
Applied Analysis

Plant Information

Overview

- The controller presented here has been tested on several plants.
- The example used for this presentation is a 500W electric kick scooter.

- Custom-designed and built hub motor.
- Rear wheel direct drive, 1:1.
- 33V, 4.4Ah LiFePO4 battery.
- Torque command by hand throttle.



Plant Information

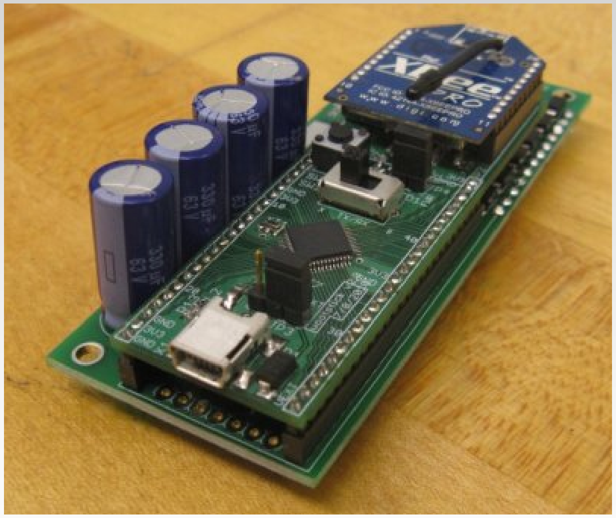
Important Specifications

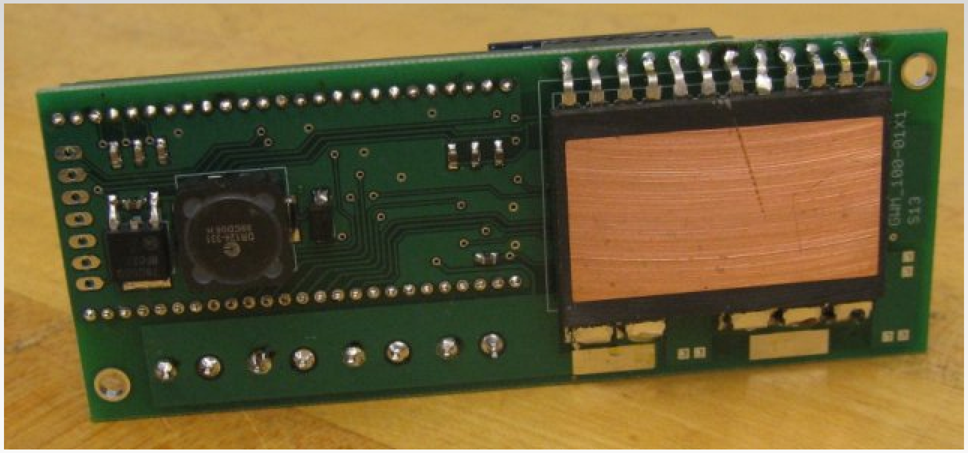
Symbol	Description	Value	Units
$2p$	Number of poles.	14	-
R_a	Per-phase motor resistance.	0.084	Ω
L_s	Synchronous inductance.	0.2×10^{-3}	H
K_t	Per-phase torque/back EMF constant.	0.10	V/(rad/s)
V	Nominal DC voltage.	33.0	V
J	Plant inertia, reflected to rotational.	0.40	$kg \cdot m^2$

Controller Hardware

Overview

- Custom 48V/40A three-phase inverter drive
- Hall effect-based current sensing (phase and DC).
- v1,2: Texas Instruments MSP430F2274 (16-bit, no hardware multiplier)
  v3: STMicroelectronics STM32F103 (32-bit, w/ hardware multiplier)
- 2.4GHz wireless link for data acquisition.





Controller Hardware

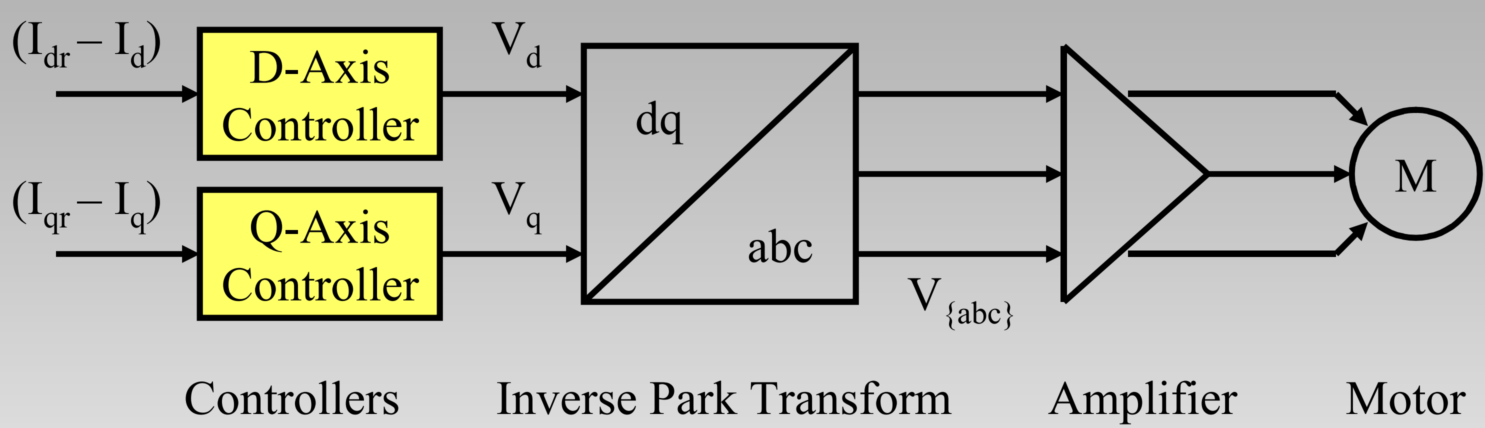
Important Specifications

Symbol	Description	Value	Units
R_{ds}	On-resistance of each phase leg.	7.5×10^{-3}	Ω
f_{sw}	PWM switching frequency.	15,625	Hz
f_{fast}	Fast-loop frequency. Handles position estimate, sine wave generation.	MSP430: 14,500 STM32: 10,000	Hz
f_{slow}	Slow-loop frequency. Handles current sampling, control computation.	MSP430: 122 STM32: 1,000	Hz
f_{tx}	Data transmit frequency. For data display and logging.	20	Hz

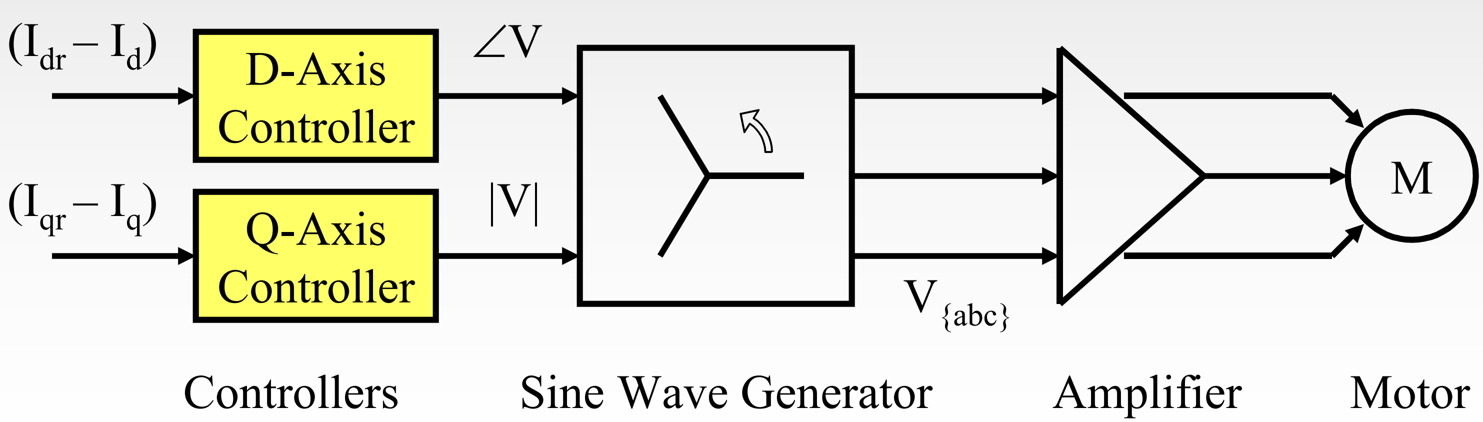
Controller Design

Overview

Synchronous Current Regulator:



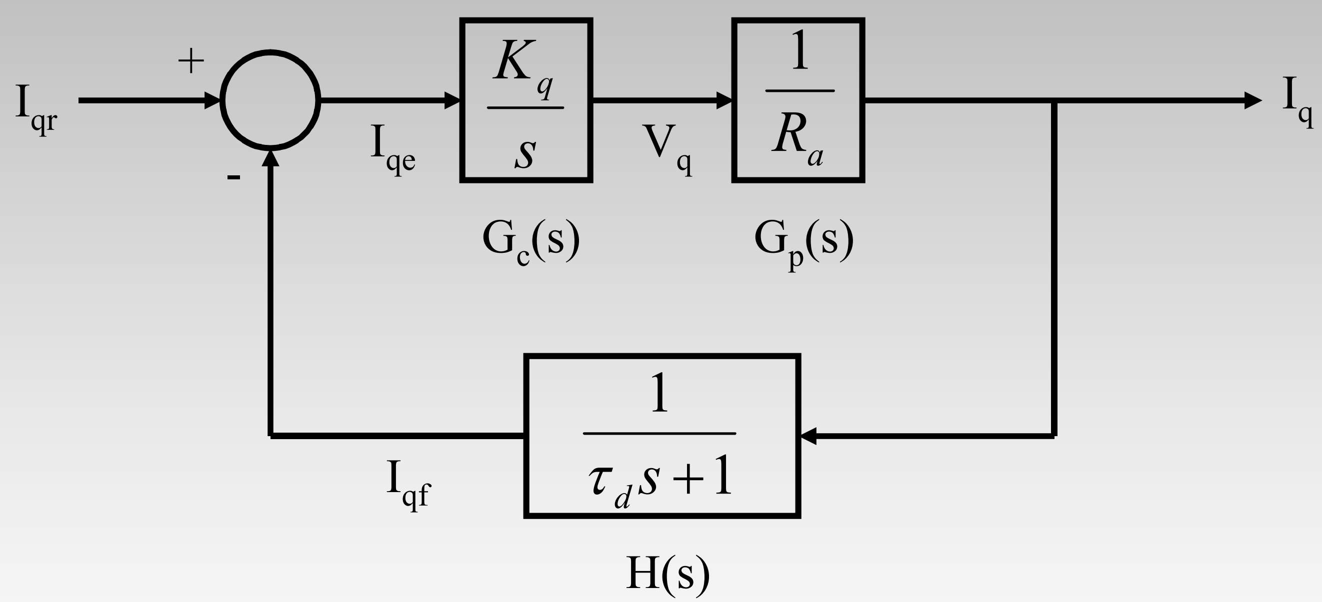
Modified Synchronous Current Regulator:



Controller Design

Simplified Plant: Q-Axis Only, Stalled

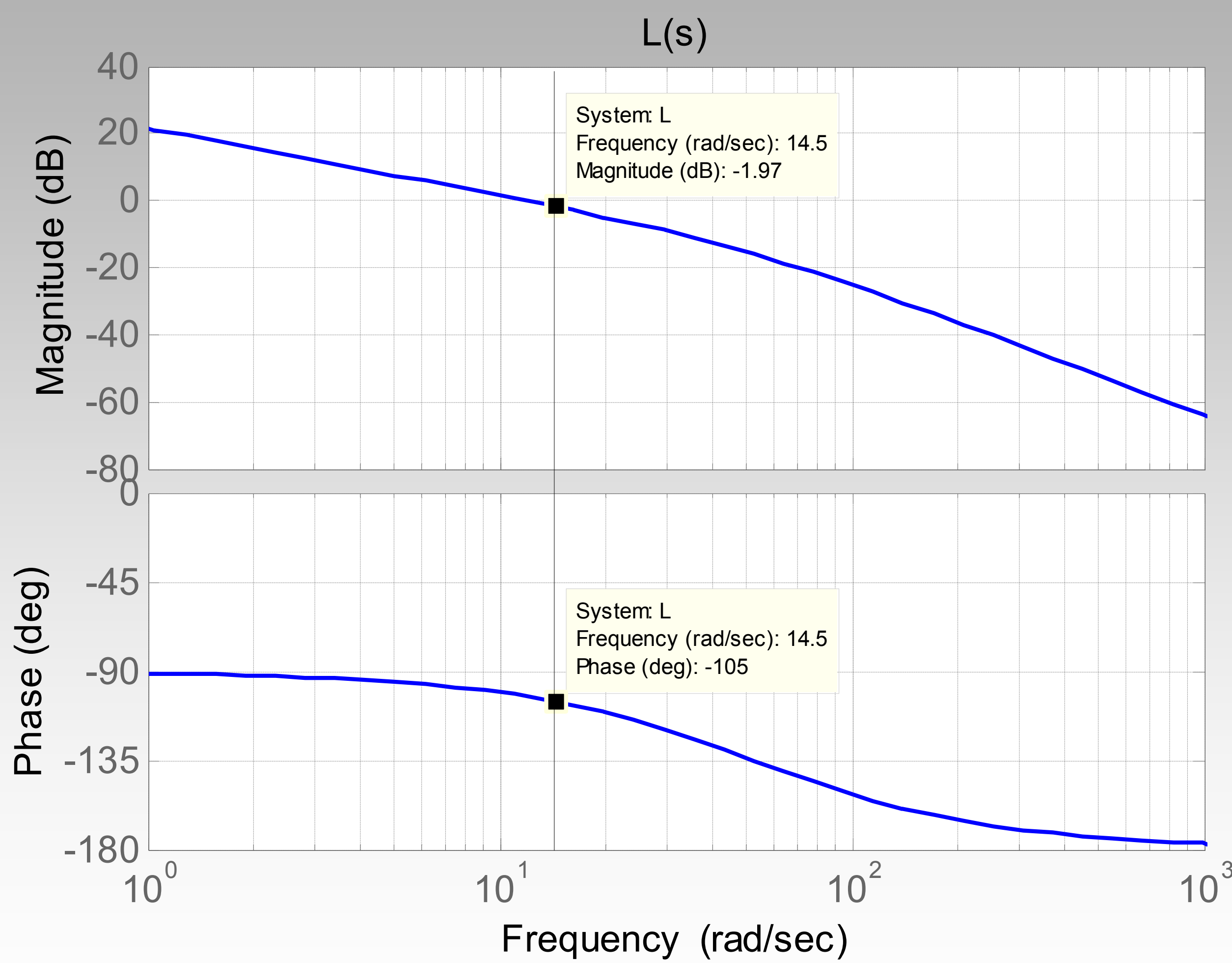
- At stall, both the d-axis and the q-axis look like resistors.
- Modeling the q-axis (torque-producing) controller and plant:



- Closed-loop poles can be placed anywhere in the left half-plane, bandwidth set by filter frequency and damping ratio set by K_q.

Controller Design

Simplified Plant: Q-Axis Only, Stalled



$$L(s) = \frac{1}{(R_a s)(\tau_d s + 1)}$$

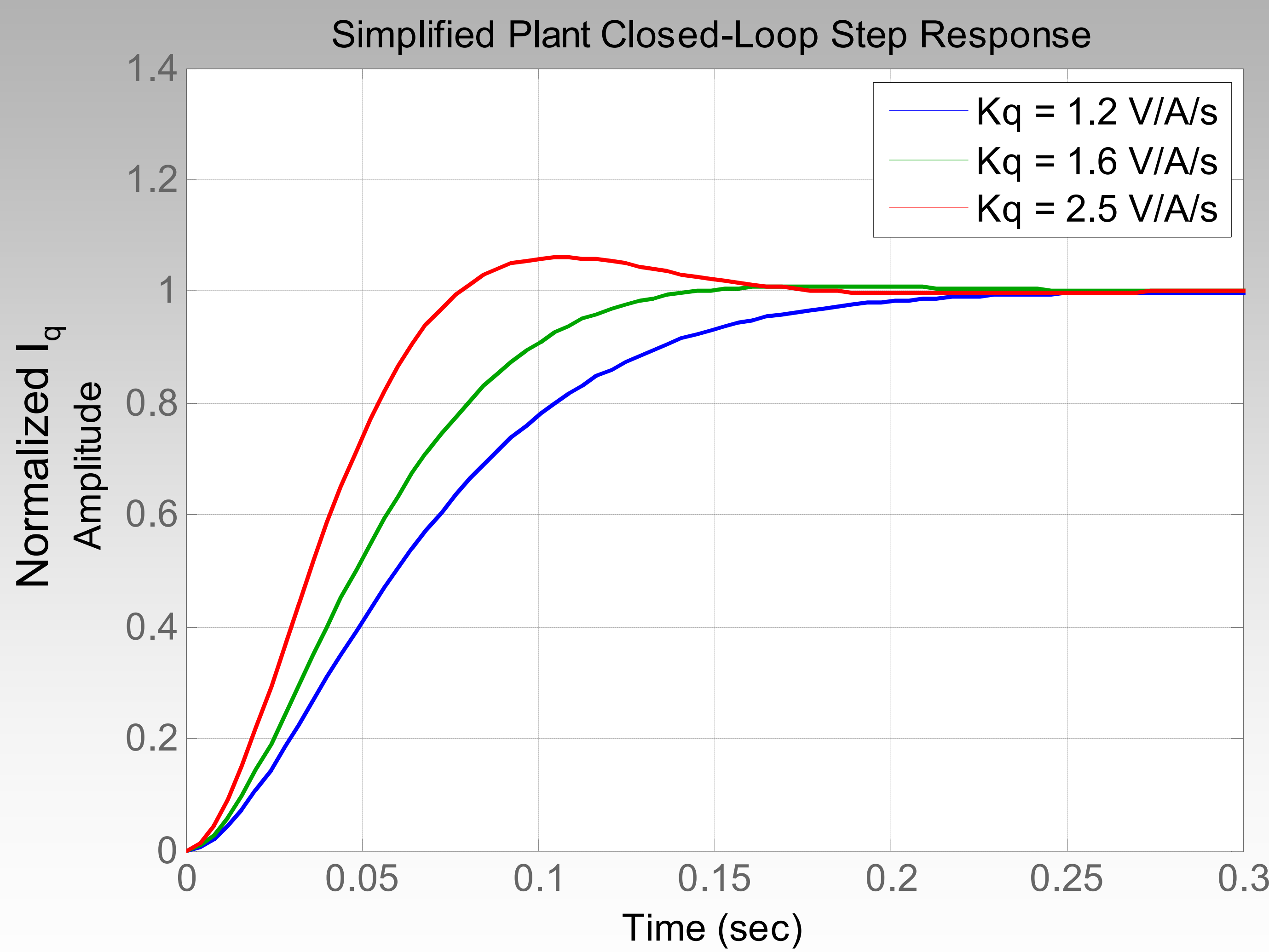
To leave 75º phase margin:

$$\boxed{K_q \approx 1.2 \frac{V}{A \cdot s}}$$

$$\phi_m = 75^\circ$$

$$\zeta \approx 0.75$$

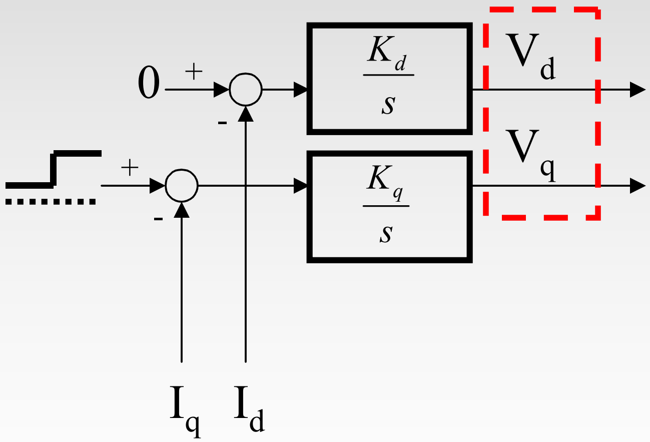
Controller Design
Simplified Plant: Q-Axis Only, Stalled



Controller Simulations

Synchronous Current Regulator

- Full motor simulation with vector quantities and complex impedance using measured motor parameters (R_a, L_s, K_t).
- Current filtering as described above.
- Speed fixed at 500rpm. (Load dynamics not considered.)
- I_{dr} = 0, I_{qr} steps from 15A to 30A.

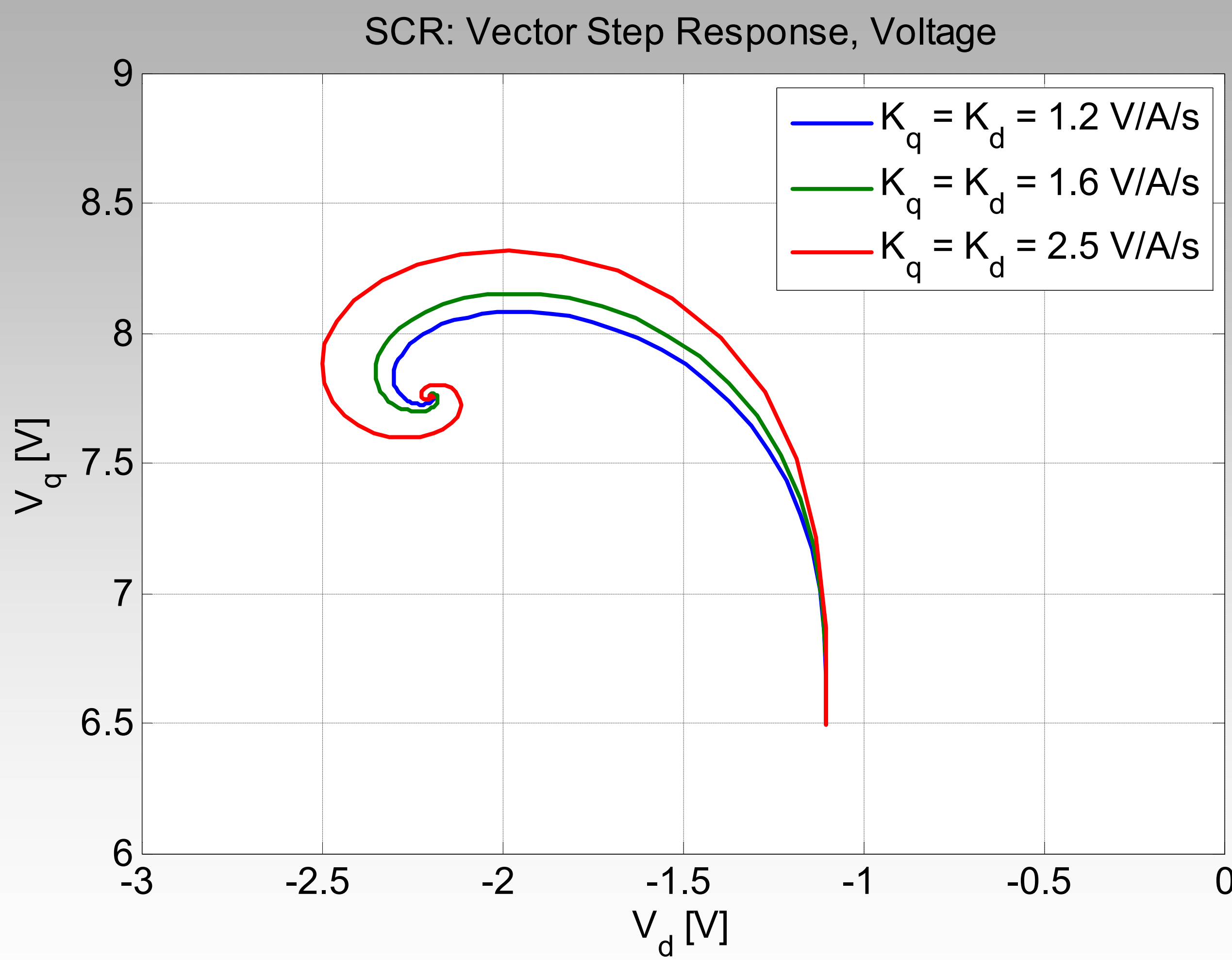


$K_d = \{1.2, 1.6, 2.5\}$ V/A/s

$K_q = \{1.2, 1.6, 2.5\}$ V/A/s

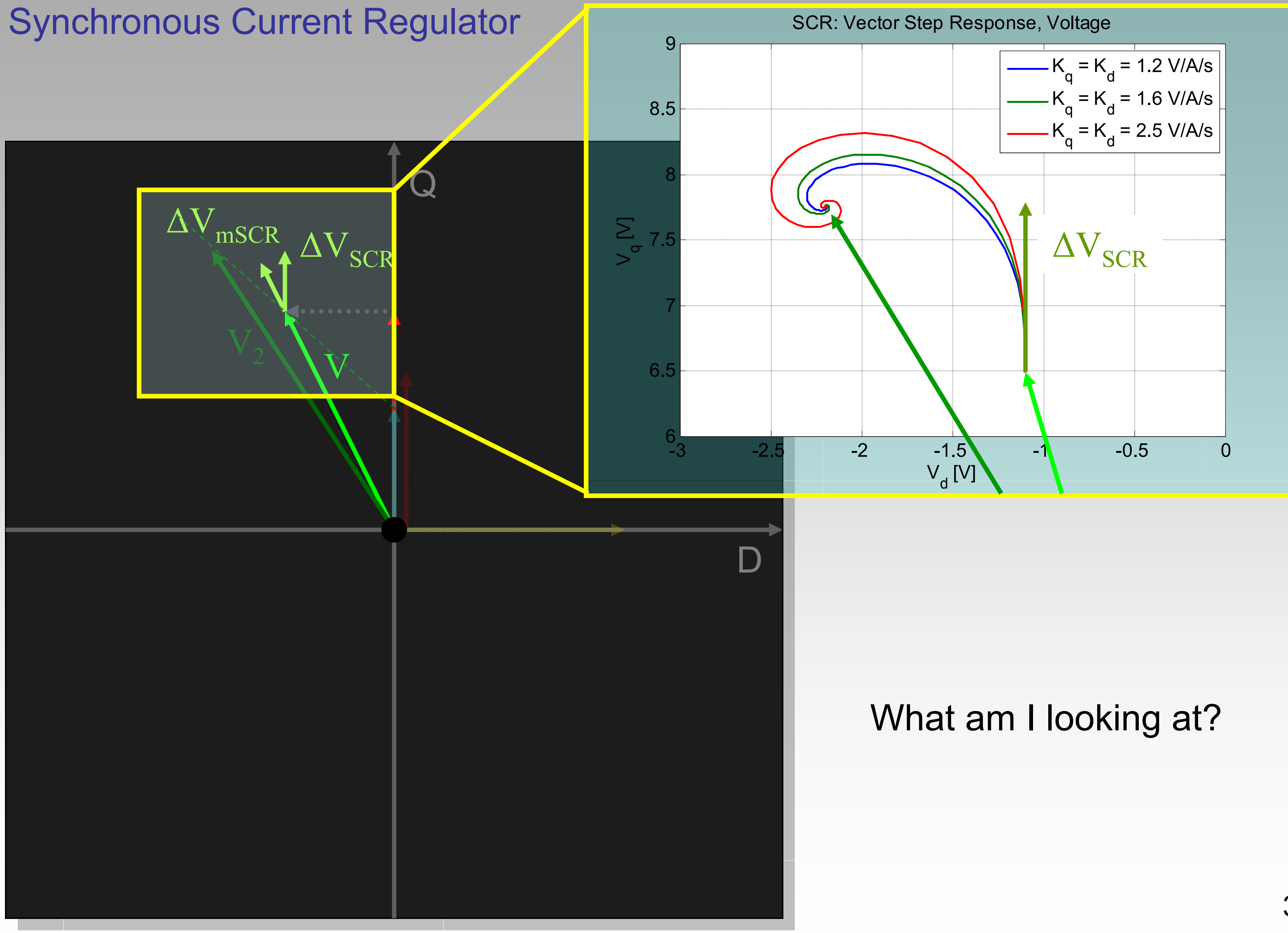
Controller Simulations

Synchronous Current Regulator



Controller Simulations

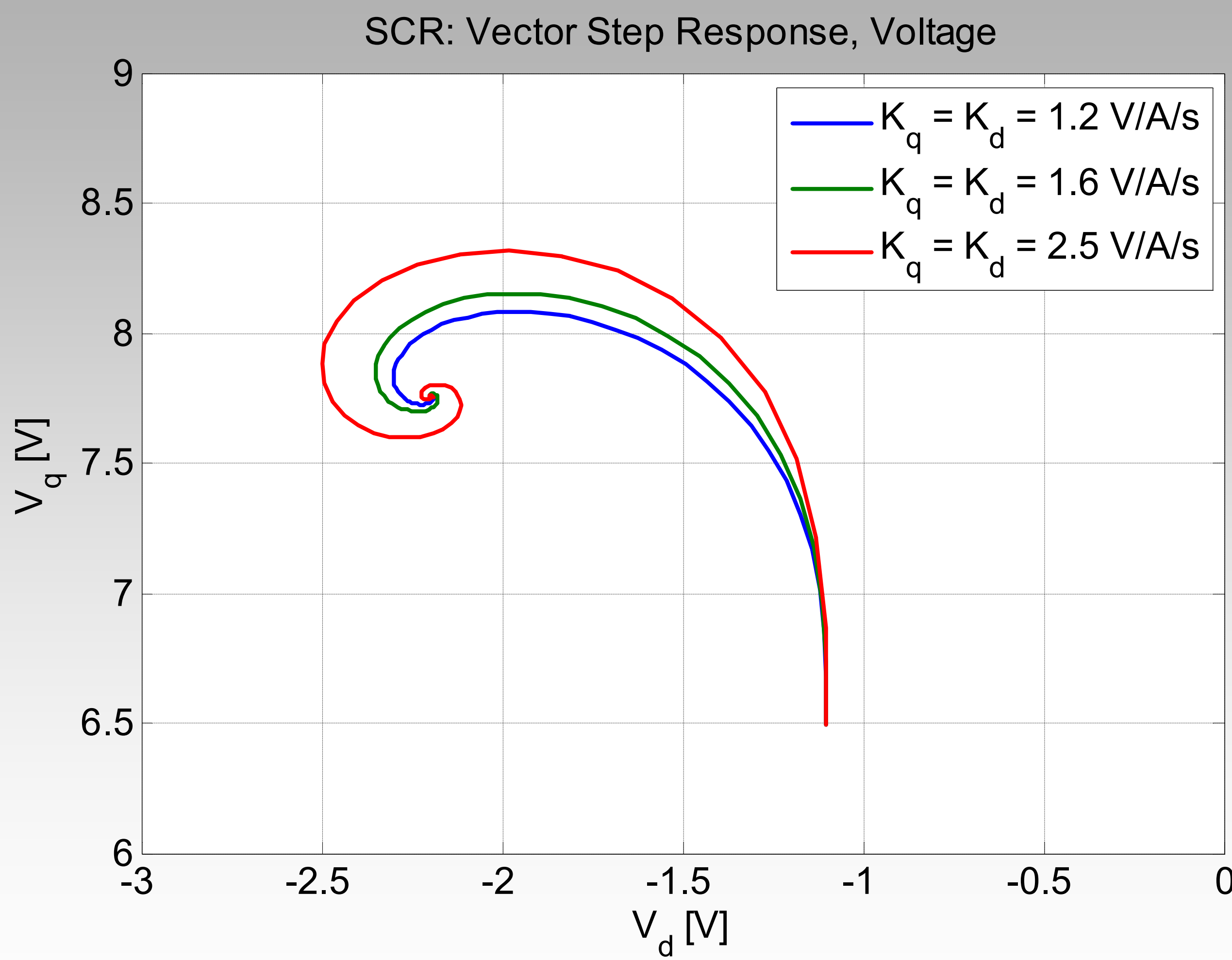
Synchronous Current Regulator



What am I looking at?

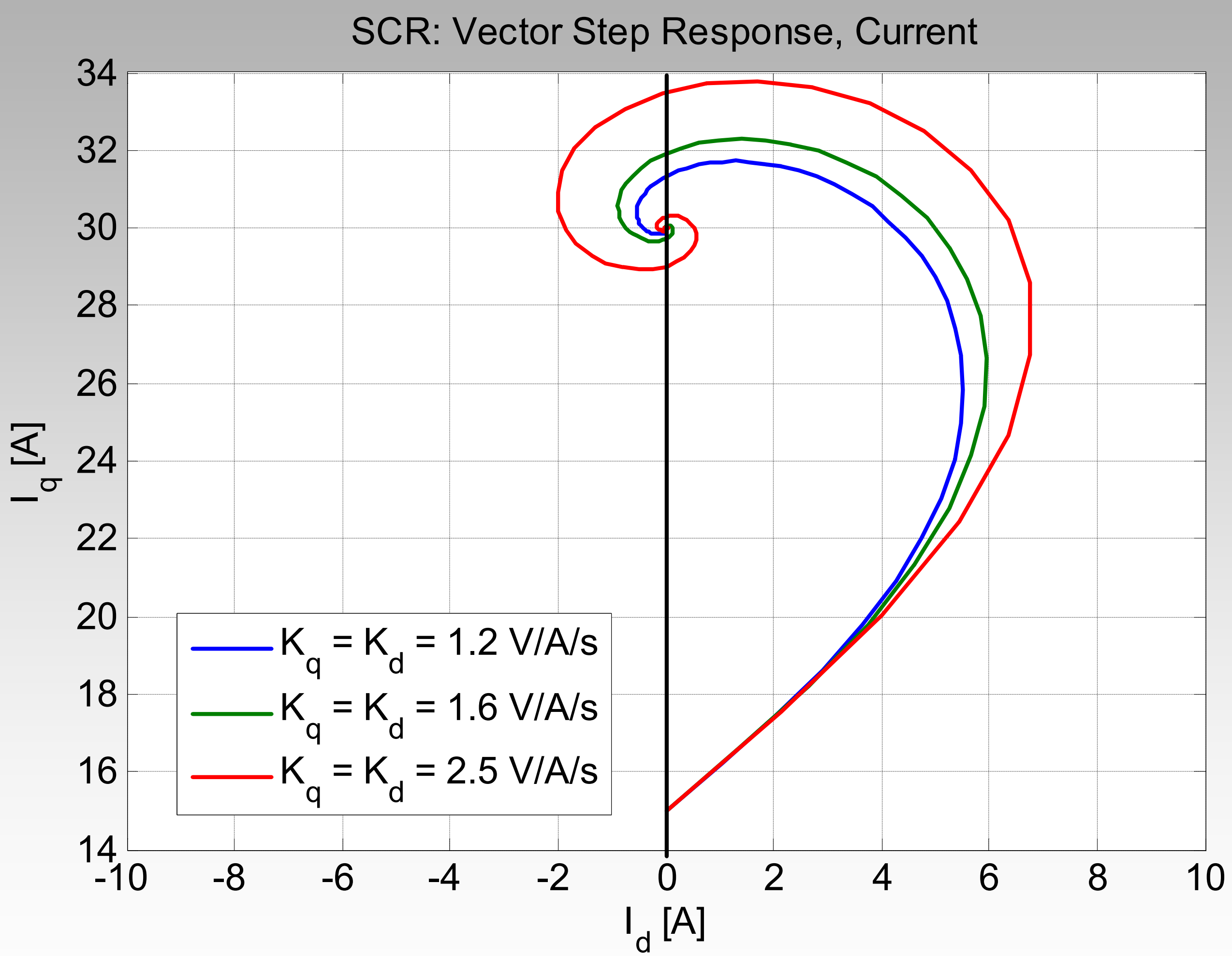
Controller Simulations

Synchronous Current Regulator



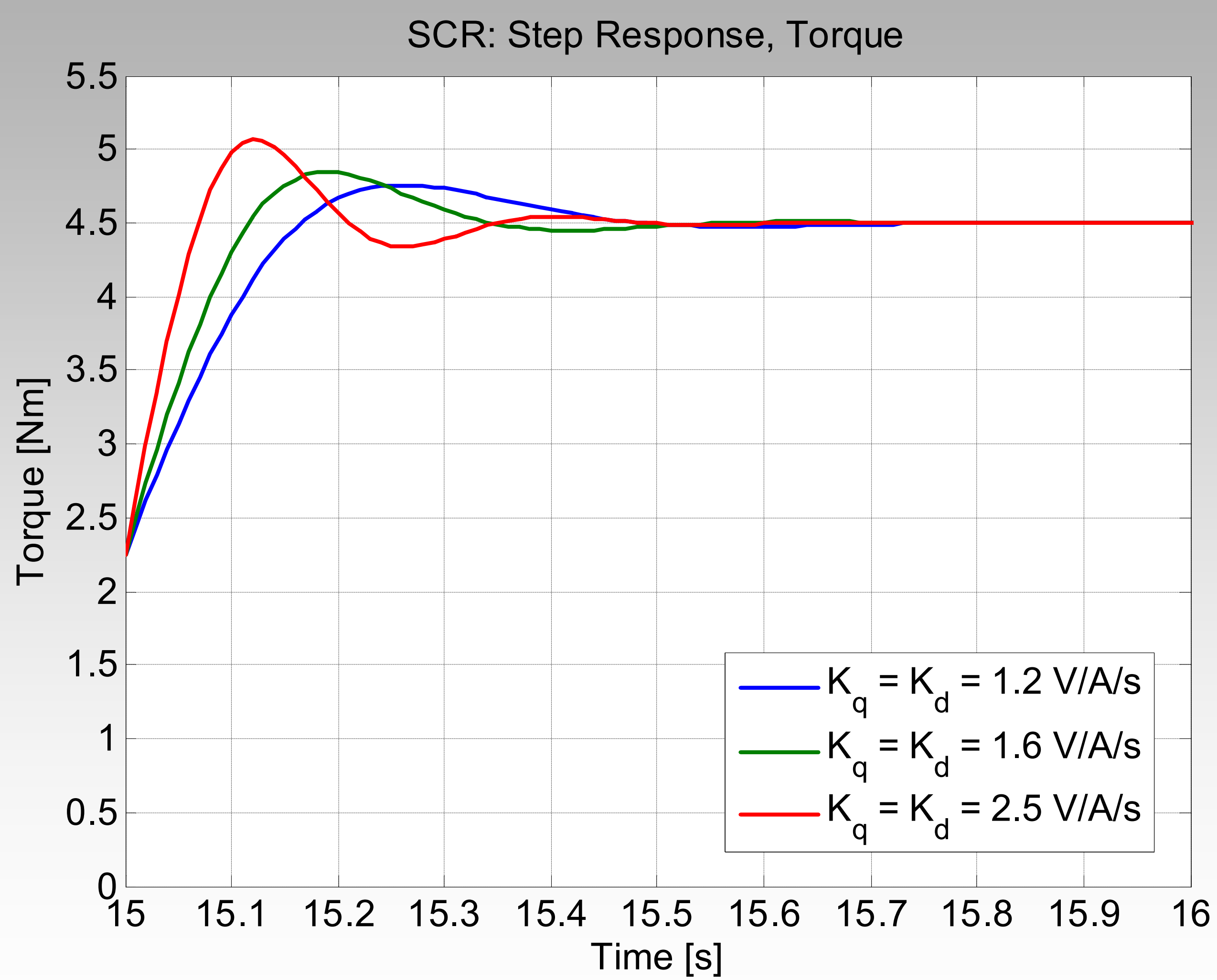
Controller Simulations

Synchronous Current Regulator



Controller Simulations

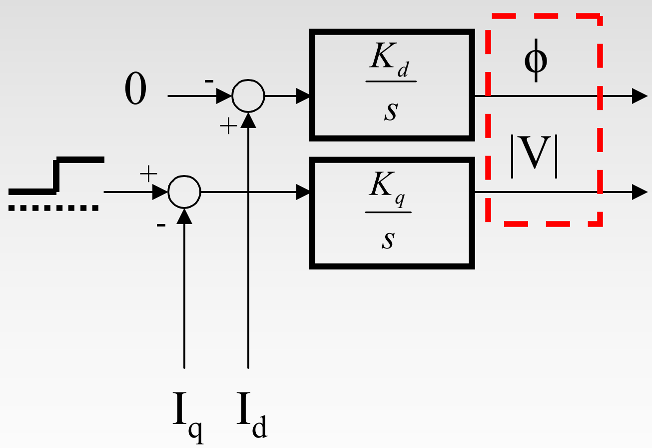
Synchronous Current Regulator



Controller Simulations

Modified Synchronous Current Regulator

- Full motor simulation with vector quantities and complex impedance using measured motor parameters (R_a, L_s, K_t).
- Current filtering as described above.
- Speed fixed at 500rpm. (Load dynamics not considered.)
- I_{dr} = 0, I_{qr} steps from 15A to 30A.

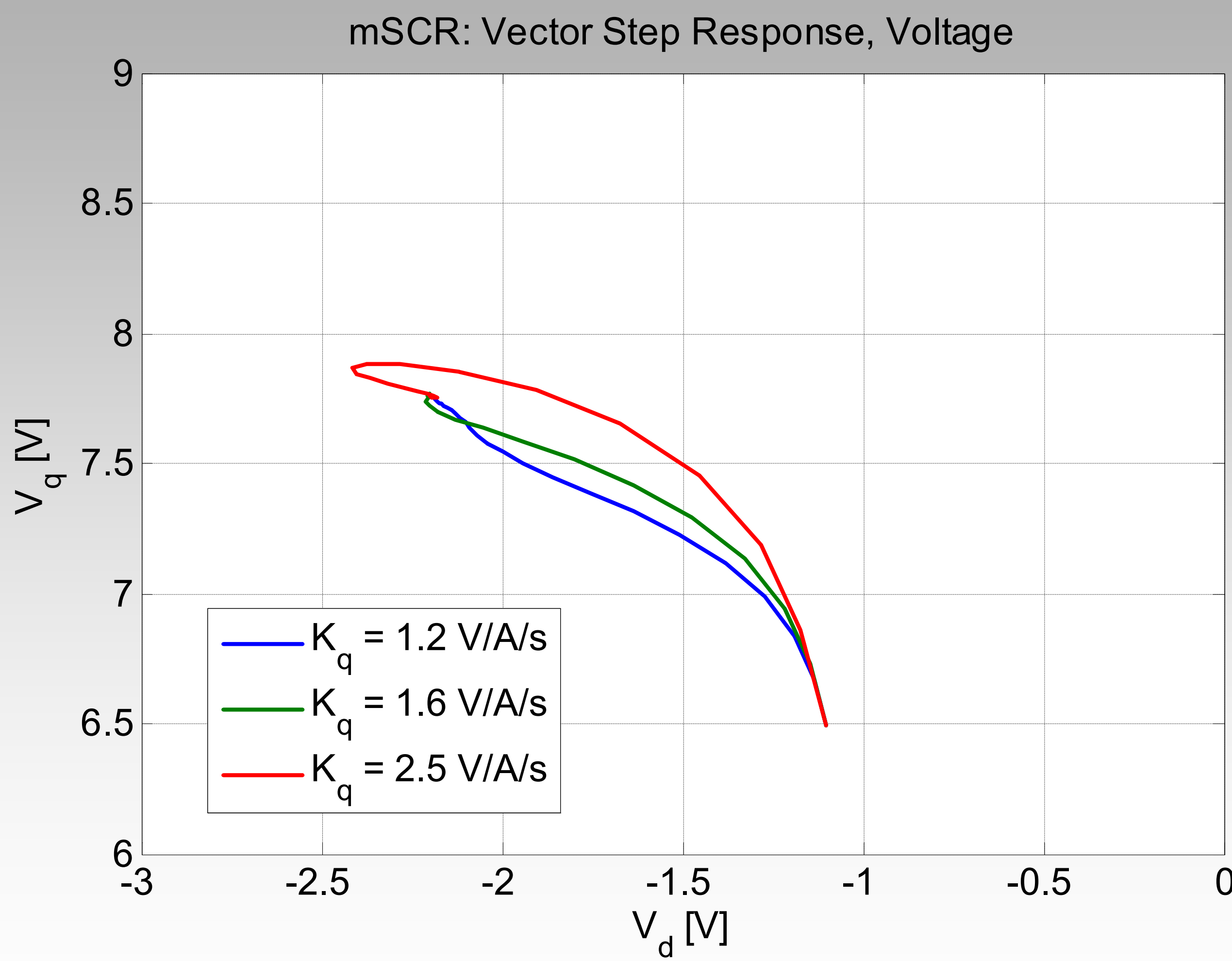


K_d = 1.0 rad/A/s (Is this fair?)

K_q = {1.2, 1.6, 2.5} V/A/s

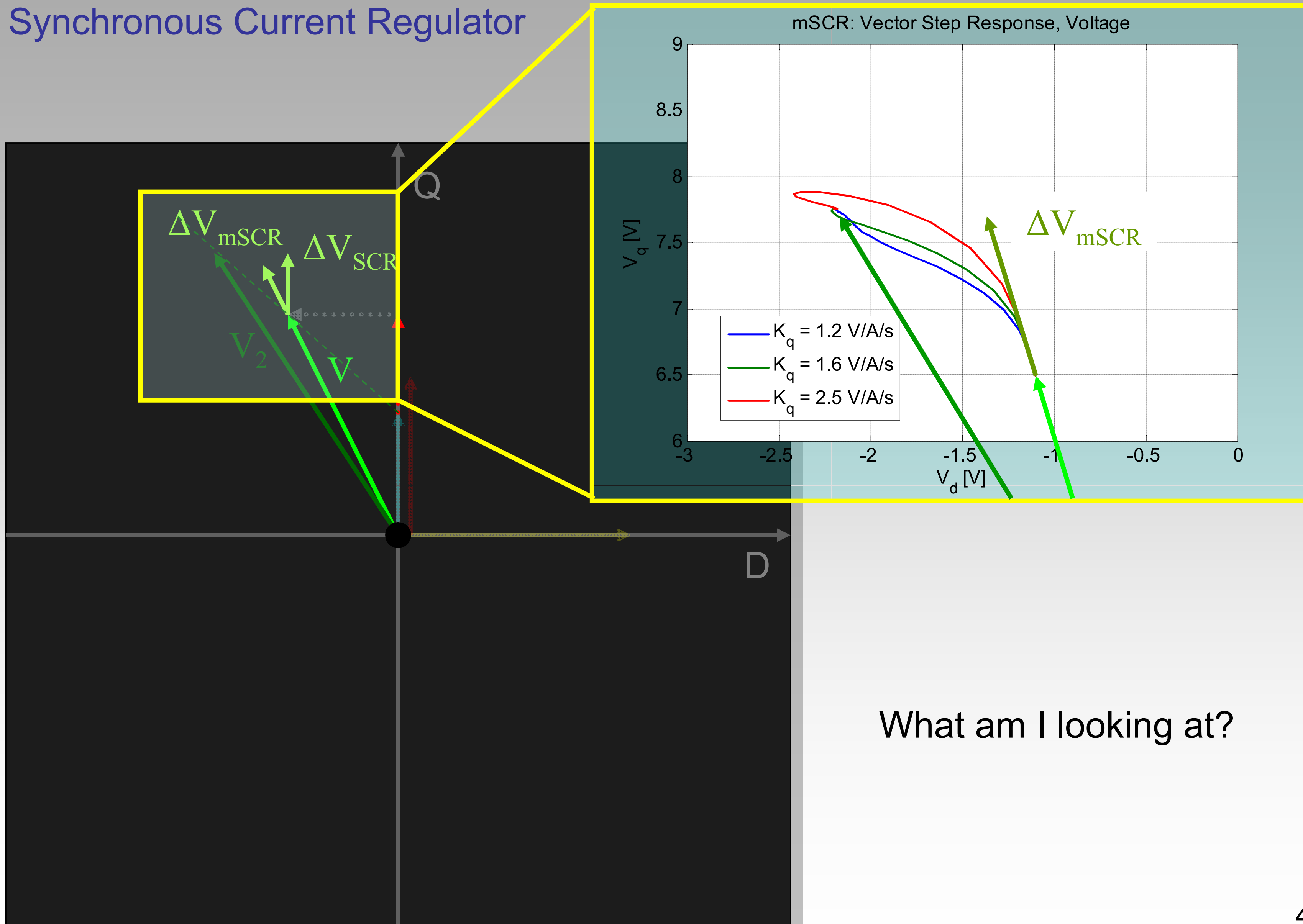
Controller Simulations

Modified Synchronous Current Regulator



Controller Simulations

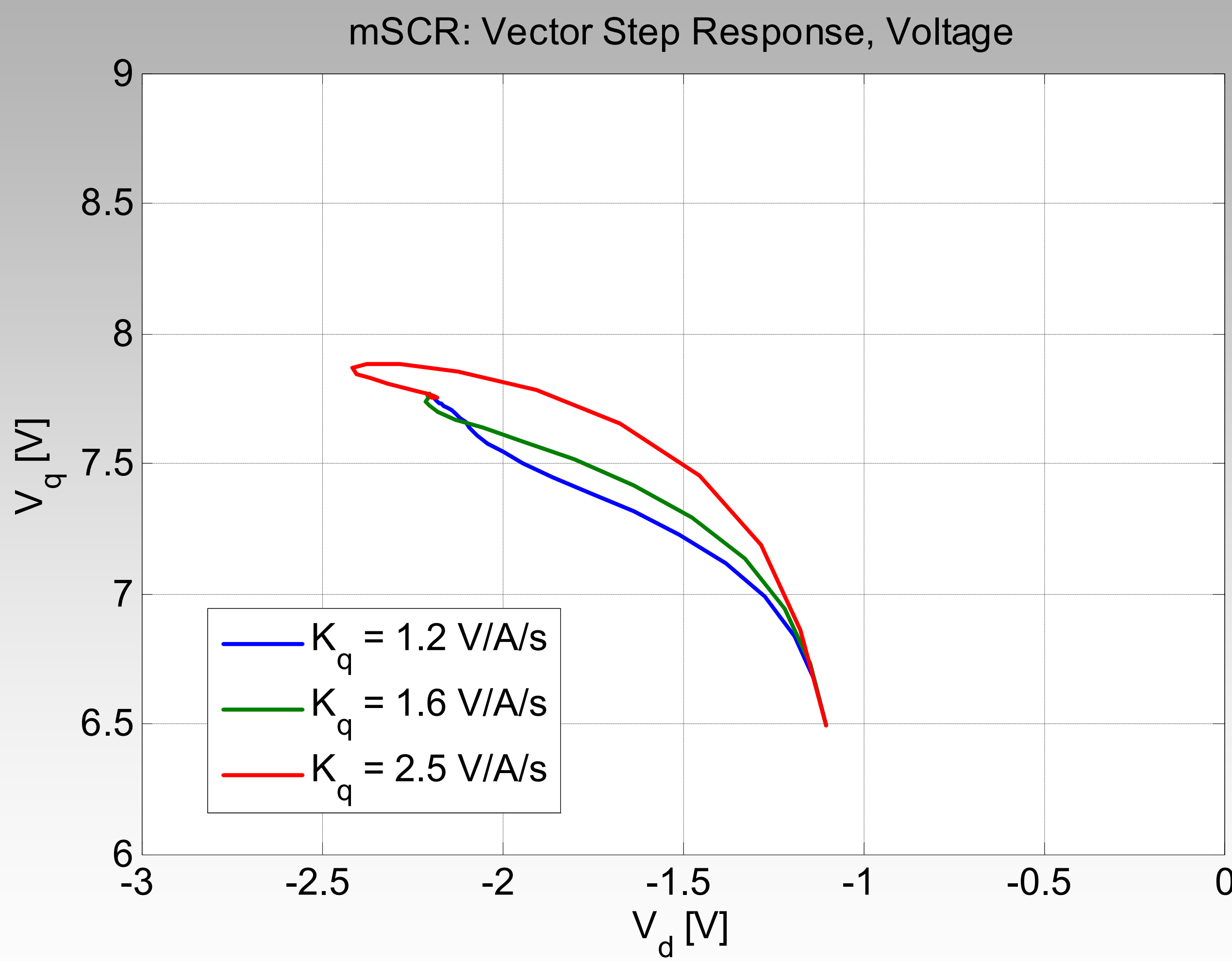
Synchronous Current Regulator



What am I looking at?

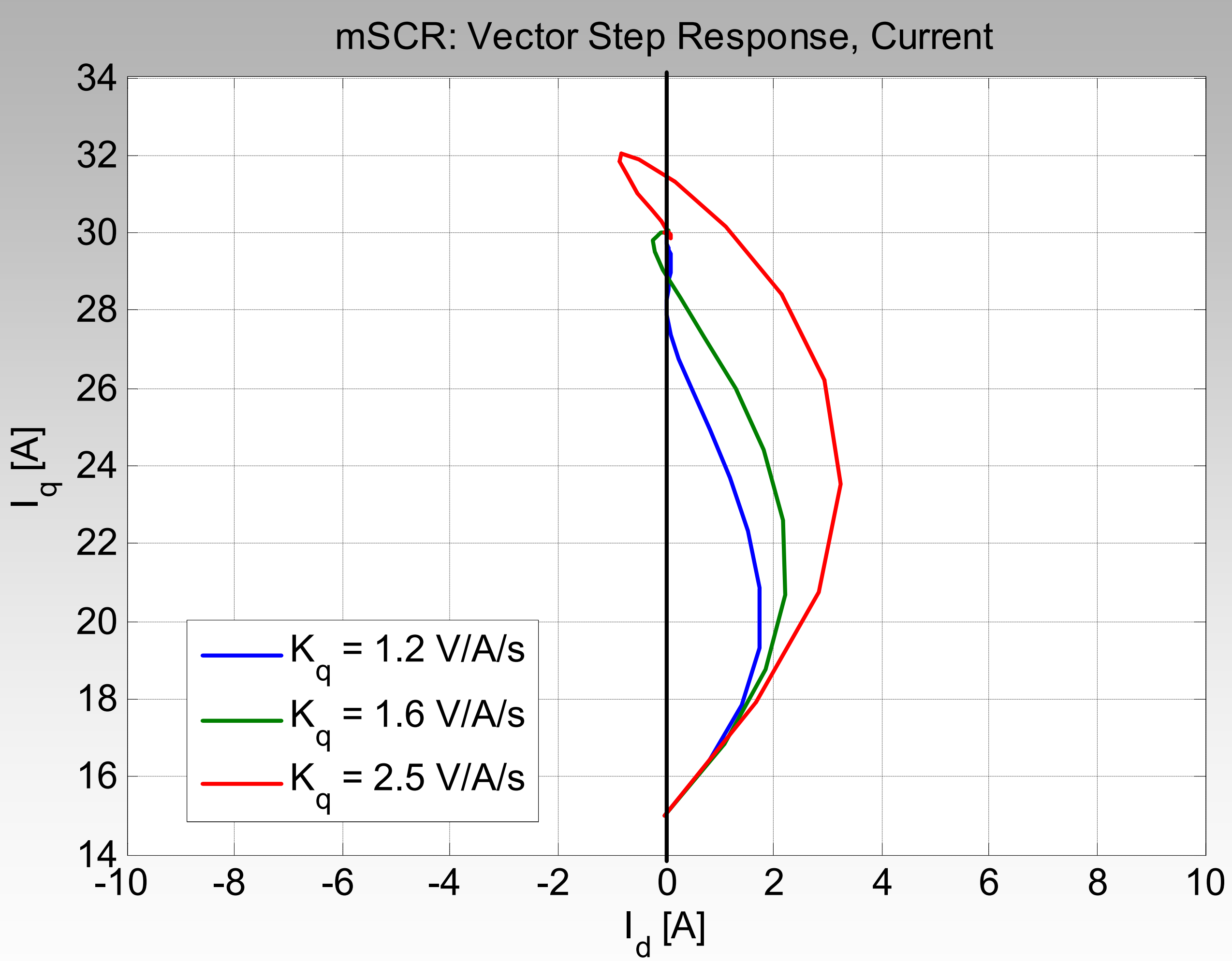
Controller Simulations

Modified Synchronous Current Regulator



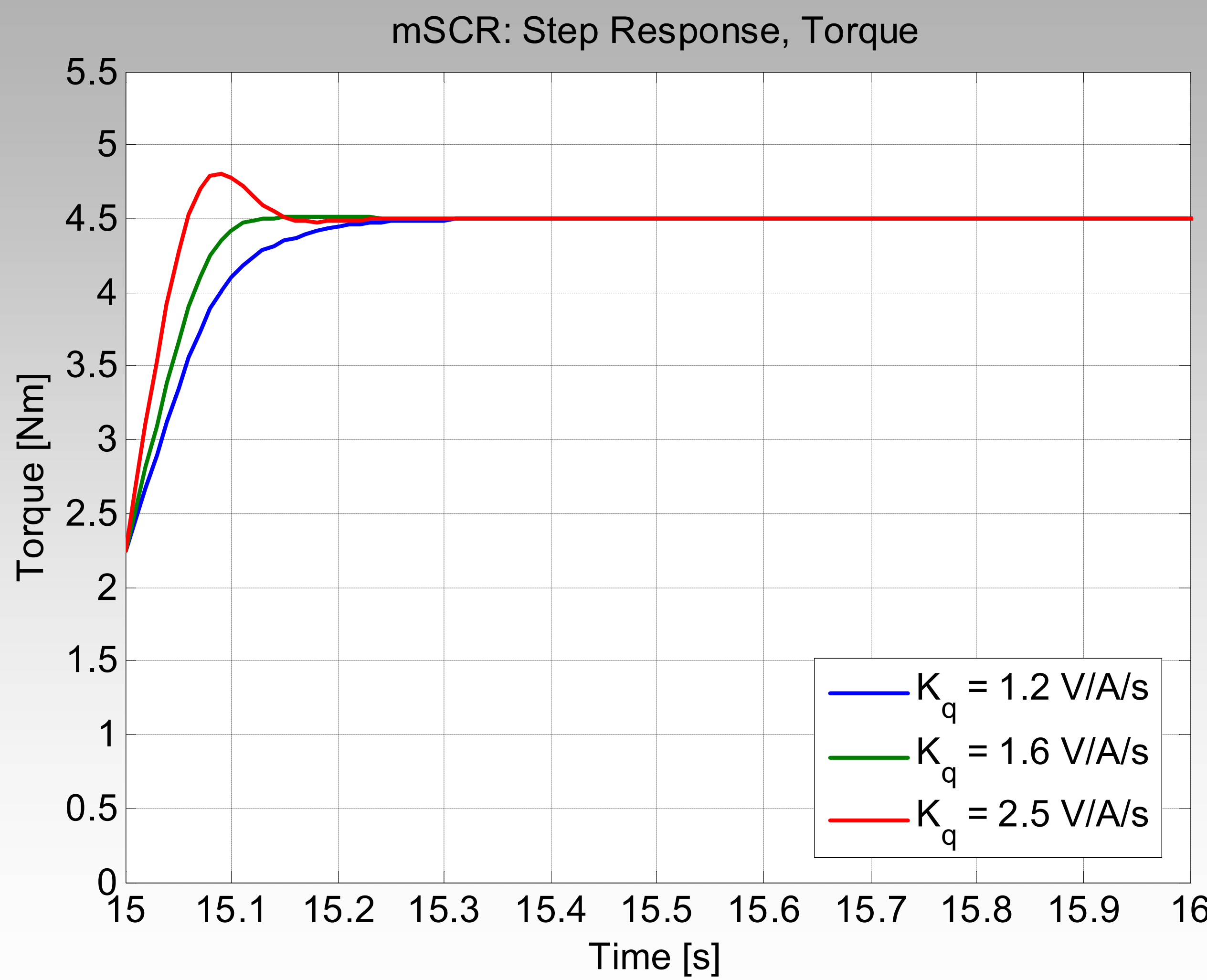
Controller Simulations

Modified Synchronous Current Regulator



Controller Simulations

Modified Synchronous Current Regulator



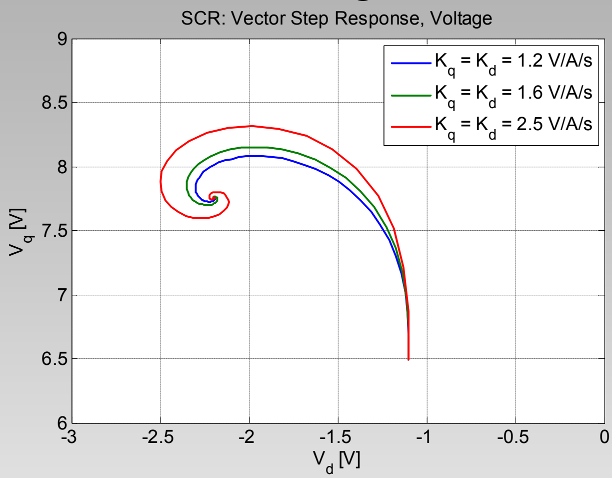
Controller Simulations

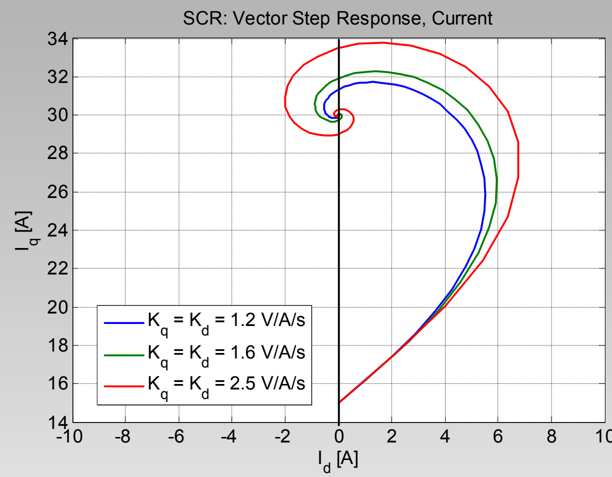
Comparison

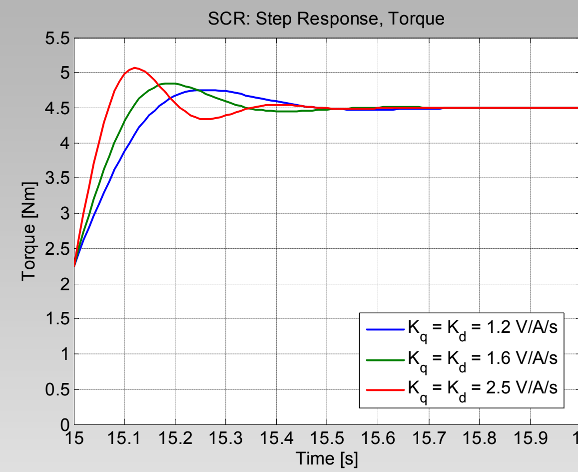
Voltage

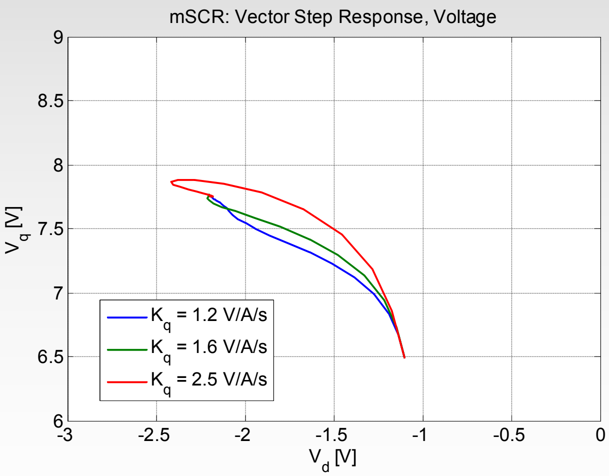
Current

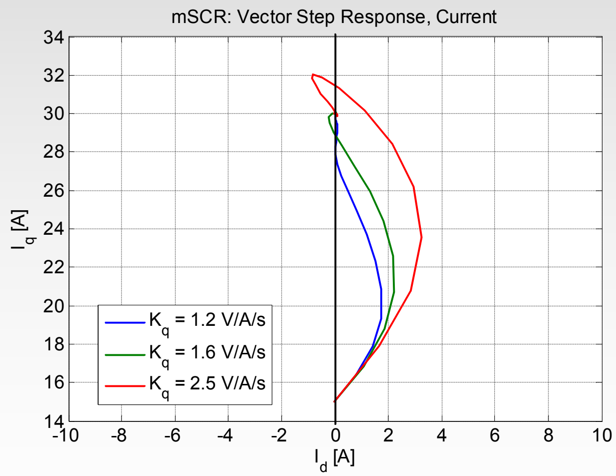
Torque

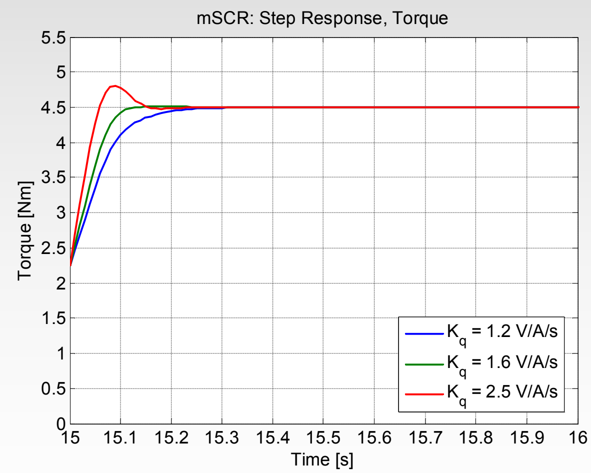






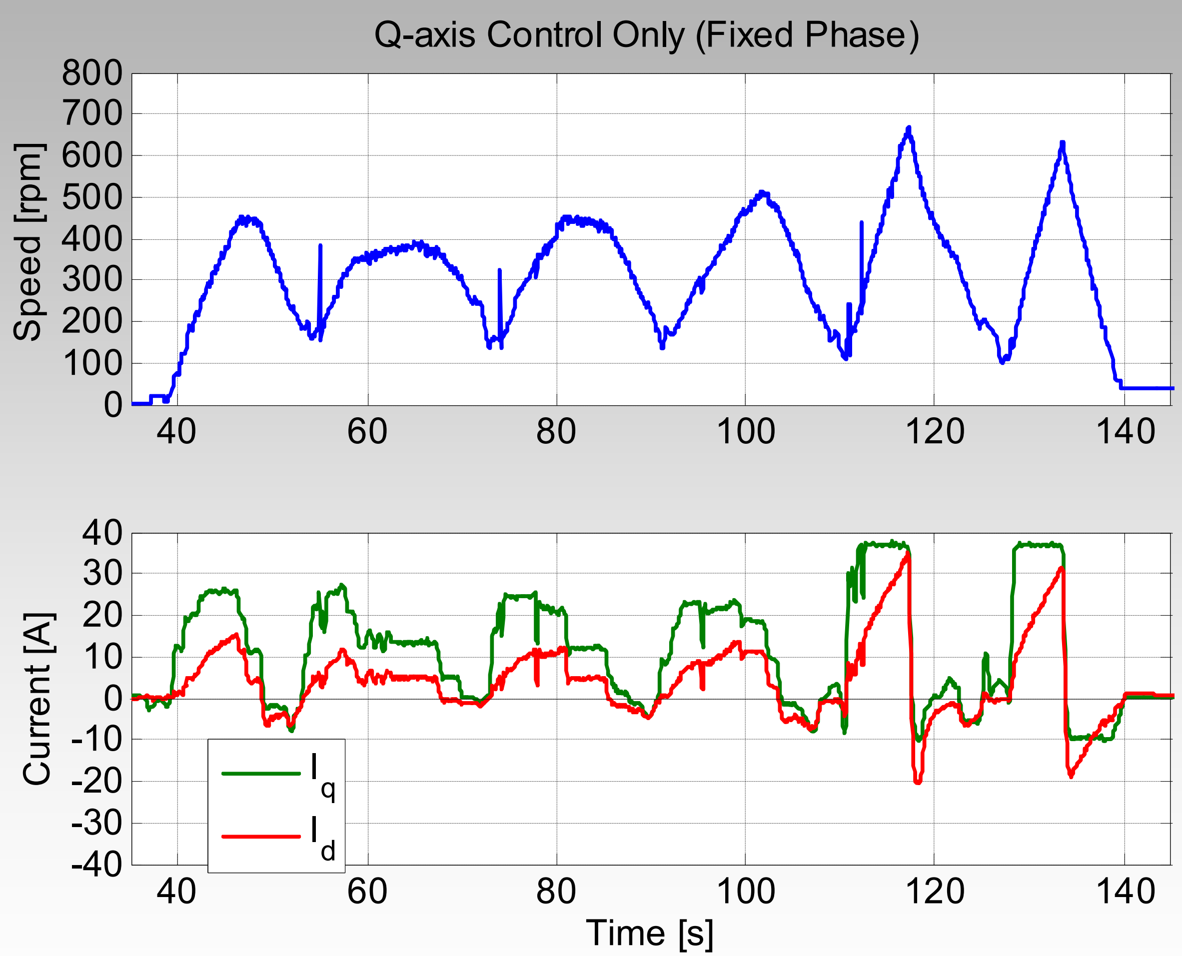






Experimental Testing and Data

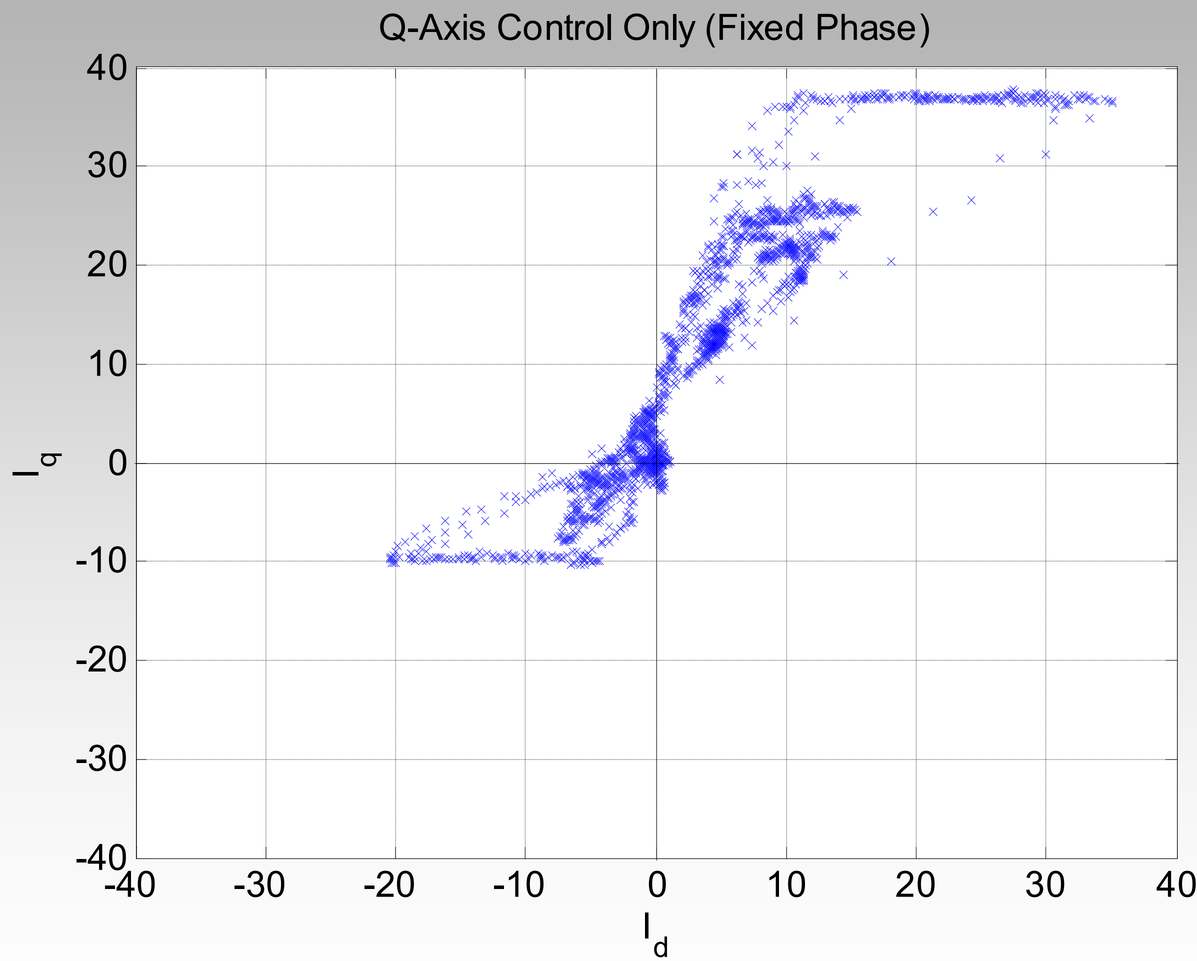
Baseline: Q-axis Control Only



- Q-axis (torque producing) current controlled.
- D-axis current increases with speed.

Experimental Testing and Data

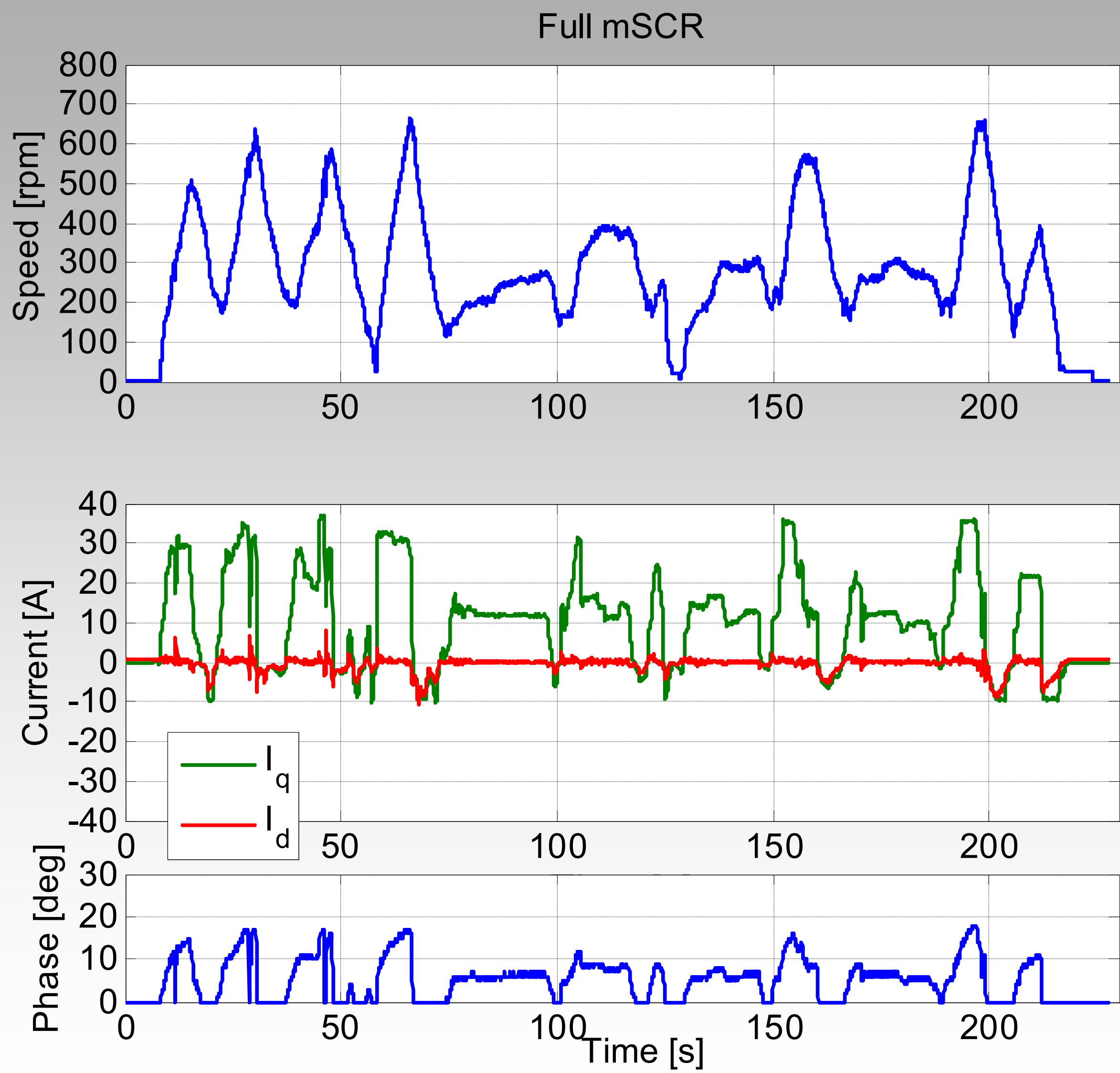
Baseline: Q-axis Control Only



- Q-axis (torque producing) current controlled.
- D-axis current increases with speed.

Experimental Testing and Data

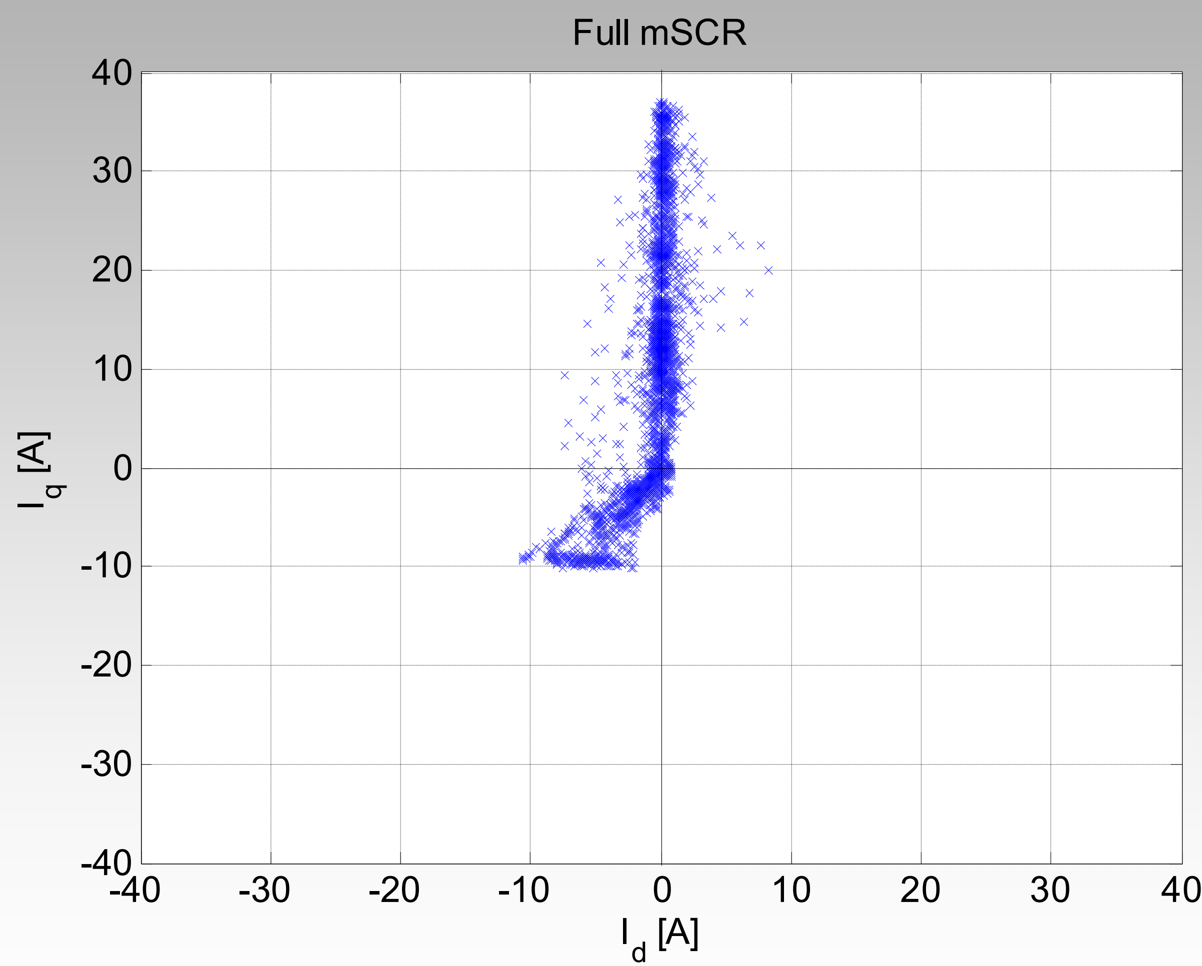
Full mSCR



- D-axis current controlled to be zero.
- Phase advanced as speed increases.

Experimental Testing and Data

Full mSCR



- In the postive torque quadrant, I_d is effectively regulated.
- Negative torque still needs work, but it's better than open-loop.

Future Work

- Range testing (or directly measure energy consumption) with SCR vs. mSCR in real-world use.
- Controlled dynamometer experiment of SCR vs. mSCR transient torque response, to verify simulations. (Requires high-speed data acquisition.)
- Sensorless control using a state observer for rotor position.
- Fault detection and recovery to increase controller robustness, possibly using sensorless control as a “back-up” in the event of sensor failure.
- More high-speed testing.
- Larger-scaled motor and controllers.

Questions / Feedback

References

[1] J.R. Mevey. *Sensorless Field Oriented Control of Brushless Permanent Magnet Motors*. M.S. Thesis. Kansas State University, Manhattan, 2009.

[2] J.L Kirtley. *Permanent Magnet 'Brushless DC' Motors*. Chapter 7 of Course Notes for 6.685 - Electric Machines. Massachusetts Institute of Technology, Cambridge, 2005.

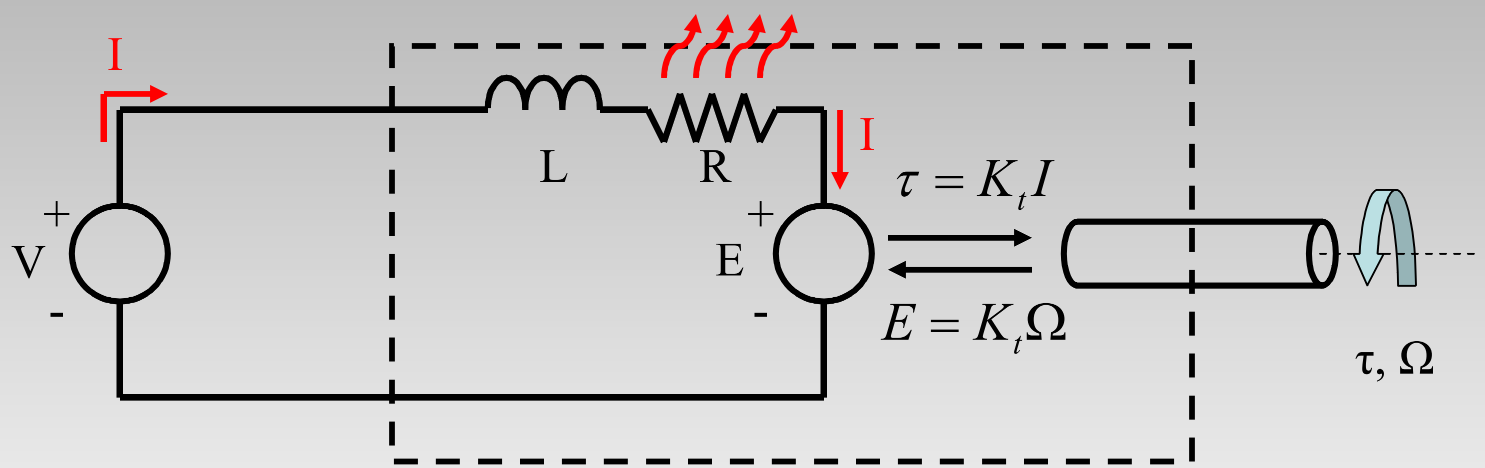
[3] A. Hughes. *Electric Motors and Drives: Fundamentals, Types, and Applications.* Third Edition. Newness, TK, 2005.

[4] T.M. Rowan, R.J. Kerkman. "A new synchronous current regulator and an analysis of current-regulated PWM inverters," IEEE Transactions on Industry Applications, vol. IA-22, no. 4, pp. 678-690, Jul./Aug. 1986.

[5] F. Briz, M.W. Degner, R.D. Lorenz. "Analysis and Design of Current Regulators Using Complex Vectors." IEEE Transactions on Industry Applications, vol. 36, no. 3, pp. 817-825, May/Jun. 2000.

Motor Control Overview

- Electric motors convert electrical power (voltage, current) to mechanical power (torque, speed), with some power lost as heat in the motor.

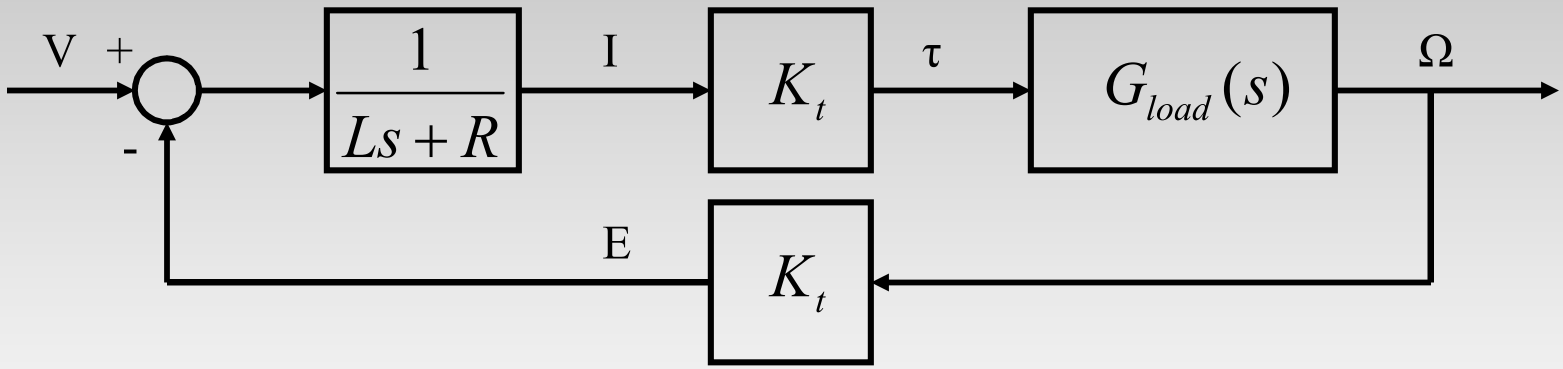


Brushed DC Motor Model

- The torque constant (K_t) and back EMF constant are identical due to power conservation. The conversion from current and back EMF to torque and speed is lossless; all losses are accounted for externally.

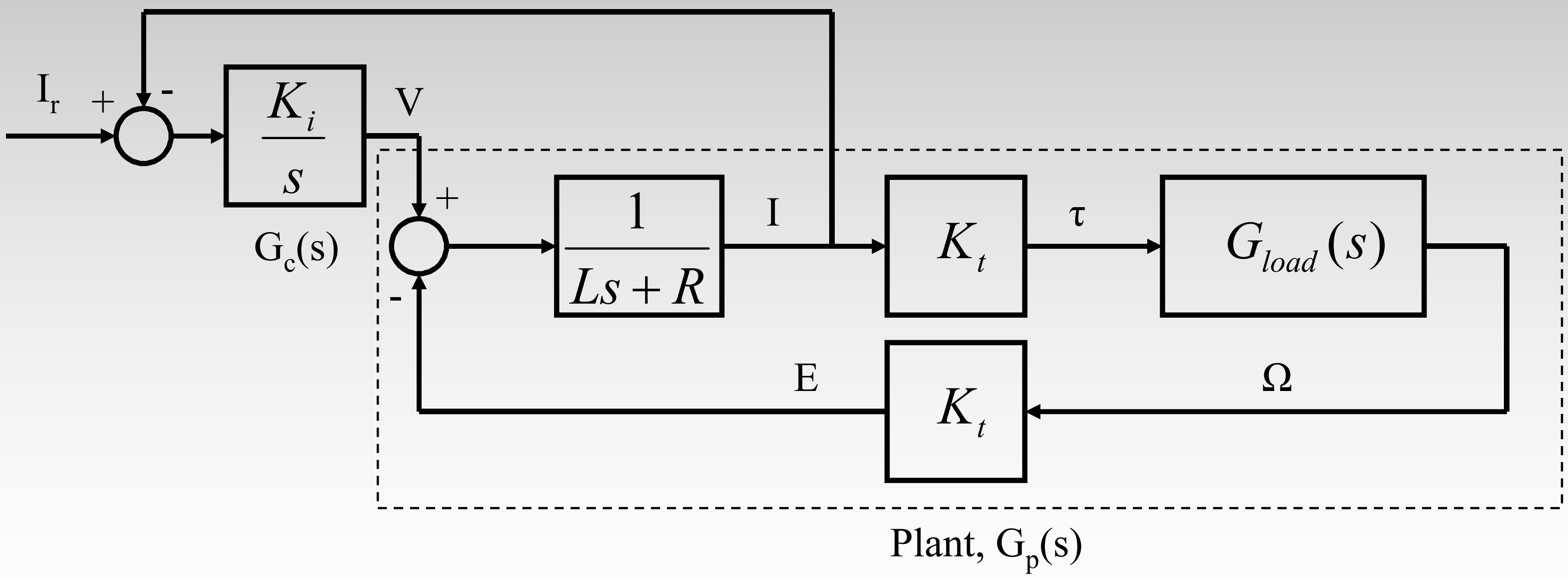
Motor Control Overview

- A *brushed* DC motor can be modeled as a SISO system (voltage to speed) with an internal feedback loop of back EMF:



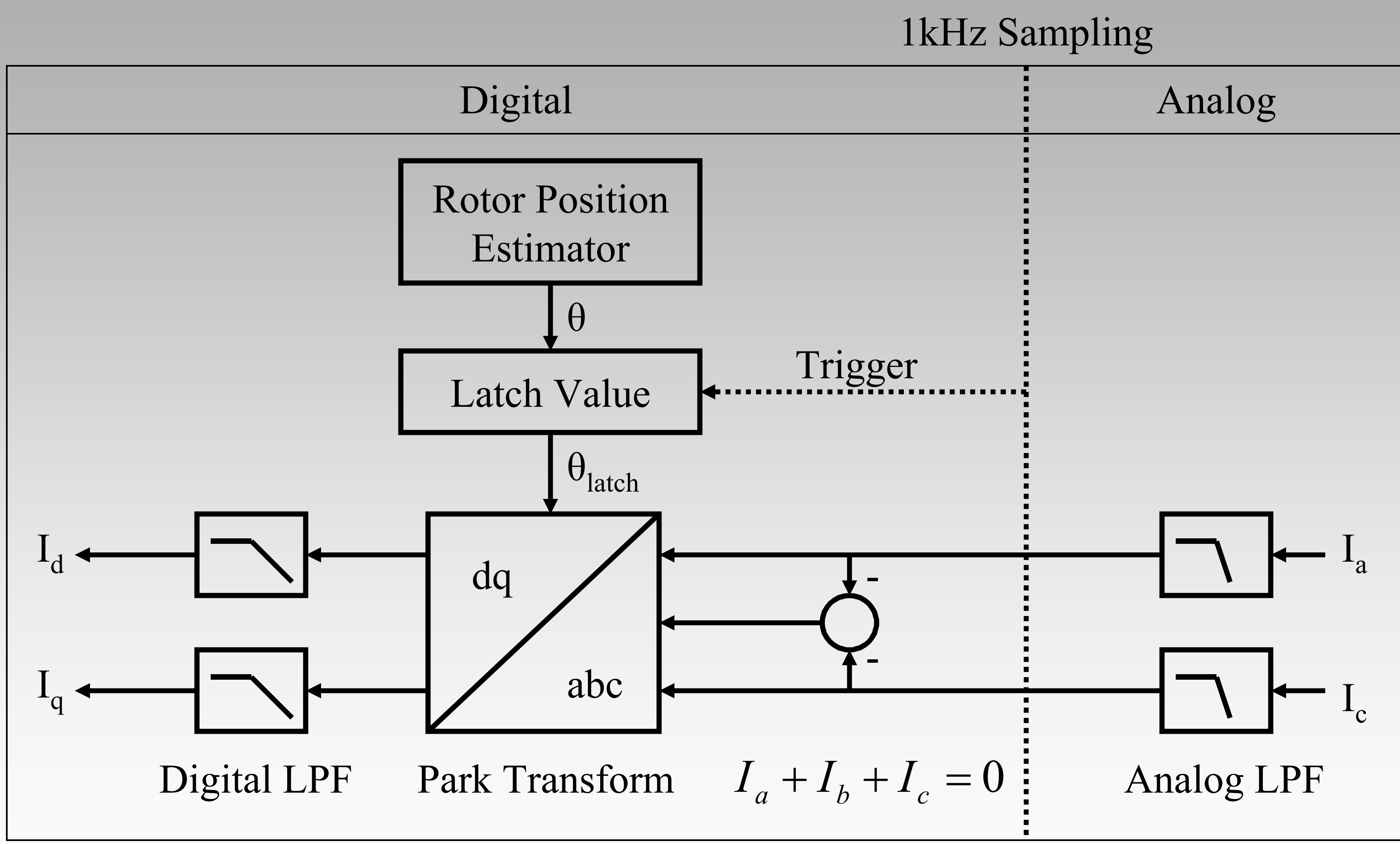
Motor Control Overview

- A current control loop provides the ability to command torque. Current is directly proportional to torque, and easy to measure.
- Depending on the load, an integral controller may be sufficient to track the reference current with zero steady-state error.



Current Sensing

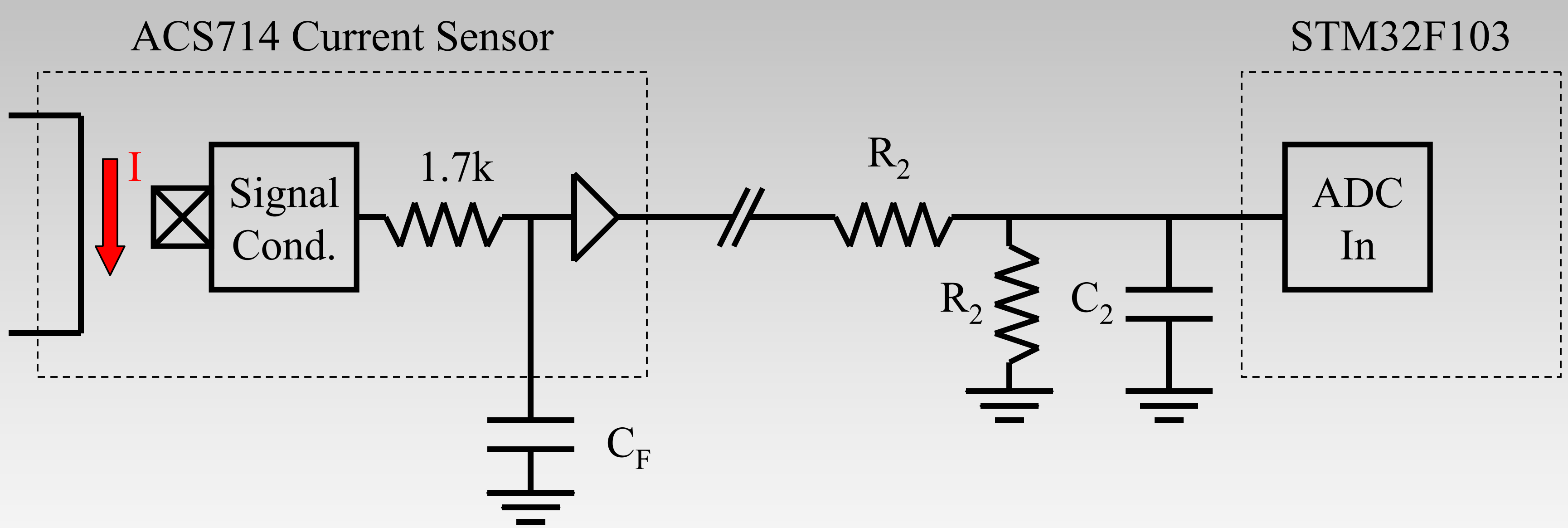
Overview



Current Sensing

Analog Filtering: Second-Order Low Pass

1. Buffered output filter on ACS714 Hall effect current sensor.
2. Local 2:1 voltage divider and RC filter at ADC pin.



$$F(s) = \left(\frac{1}{\tau_1 s + 1}\right)\left(\frac{1}{\tau_2 s + 1}\right)$$

$$\tau_1 = (1.7k\Omega)(C_F)$$

$$\tau_2 = \tfrac{1}{2} R_2 C_2$$

Current Sensing

Analog Filtering: Second-Order Low Pass

- The goal is to do as little filtering of the AC current signal as possible, so as not to distort the phase of the current. (Less than 5º phase lag desireable.)
- The PWM frequency (15,625Hz) is an obvious target for filtering.
  1. Actual current ripple will be at this frequency.
  2. Power transient-induced noise will be here, too.
- The filtering after the Park Transform can be much more aggressive, so noise in the AC current signal is acceptable.
- Component Selection:

$$C_F = 10nF$$

$$R_2 = 10k\Omega$$

$$C_2 = 10nF$$

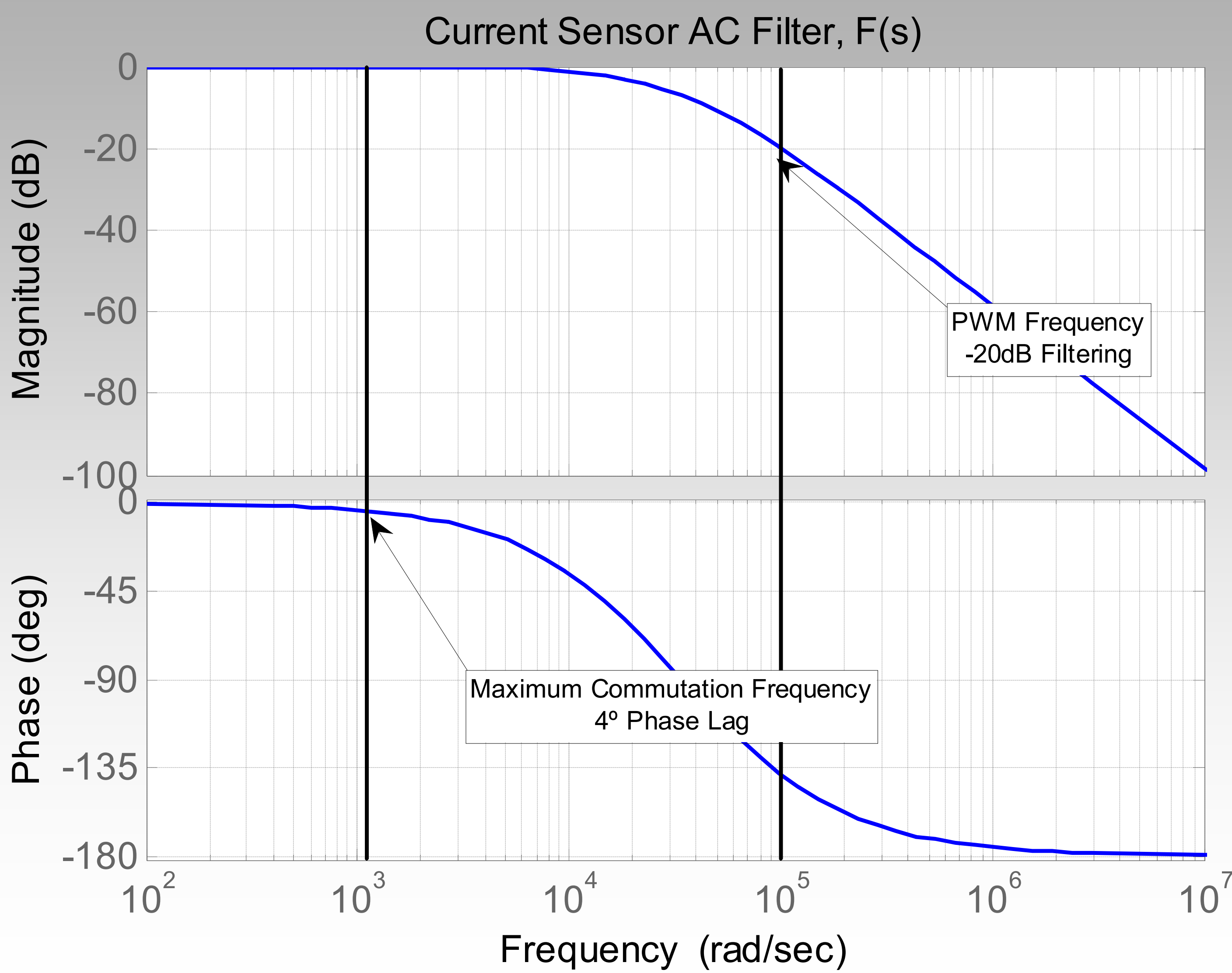
$$\tau_1 = (1.7k\Omega)(10nF) = 17\mu s$$

$$\tau_2 = \tfrac{1}{2}(10k\Omega)(10nF) = 50\mu s$$

$$\boxed{F(s) = \left(\frac{1}{\tau_1 s + 1}\right)\left(\frac{1}{\tau_2 s + 1}\right)}$$

Current Sensing

Analog Filtering: Second-Order Low Pass



Current Sensing

Digital Filtering: First-Order Low Pass

- The digital filter acts on I_d and I_q, the outputs of the Park transform.
- At steady-state, these are DC quantities. *The filter time constant can be much slower than the commutation frequency*.
- The bandwidth lower limit is driven by the target performance of the current (torque) controller.
- The bandwidth upper limit is driven by the sampling frequency. The filter time constant should be much longer than the sampling interval.
- Where Δt is the sampling interval, a first-order digital low pass filter on I_d and I_q can be implemented with the following difference equations:

$$I_q^n = a \cdot I_q^{n-1} + (1-a) \cdot I_q'$$

$$I_d^n = a \cdot I_d^{n-1} + (1-a) \cdot I_d'$$

Equivalent continuous time constant:

$$\tau_d = \frac{a}{1-a} \Delta t$$

Current Sensing
Digital Filtering: First-Order Low Pass

- Parameter Selection:

$$\Delta t = 1ms \qquad a = 0.95$$

$$\tau_d = \left(\frac{a}{1-a} \Delta t \right) = \left(\frac{0.95}{0.05} 1ms \right) = 19ms$$

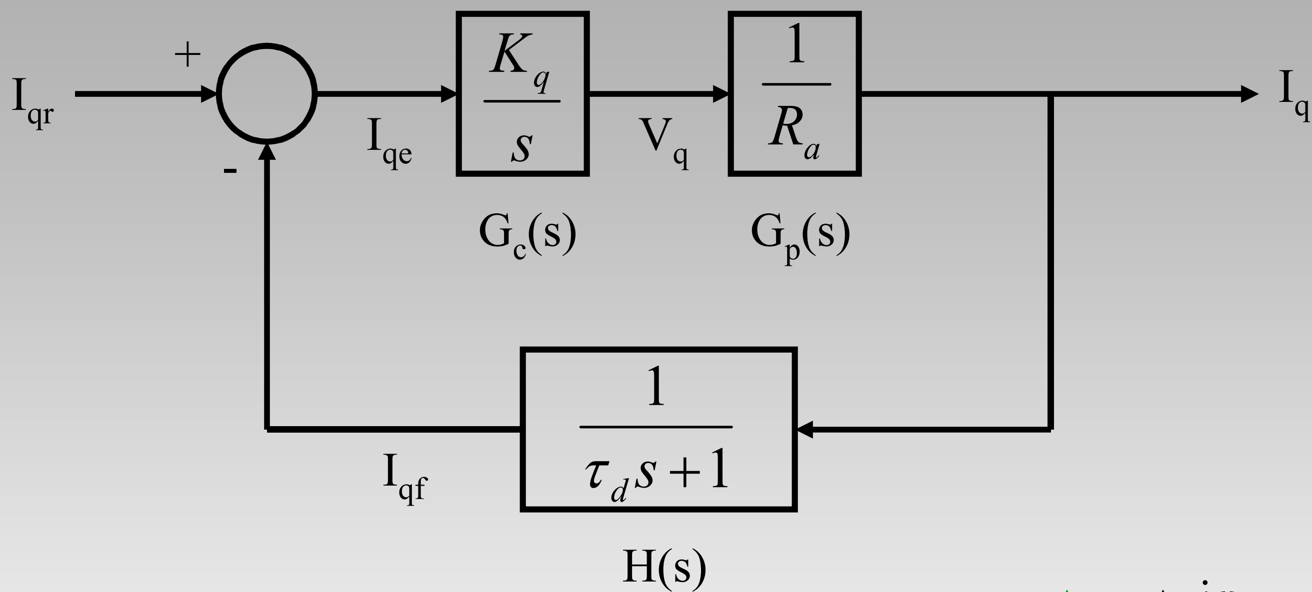
- The filter time constant is significantly longer than the sampling interval, so a “continuous time” analysis is appropriate:

$$H(s) \approx \frac{1}{\tau_d s + 1}$$

- The bandwidth is $1/\tau_d$, 52.6rad/s, or 8.38Hz.

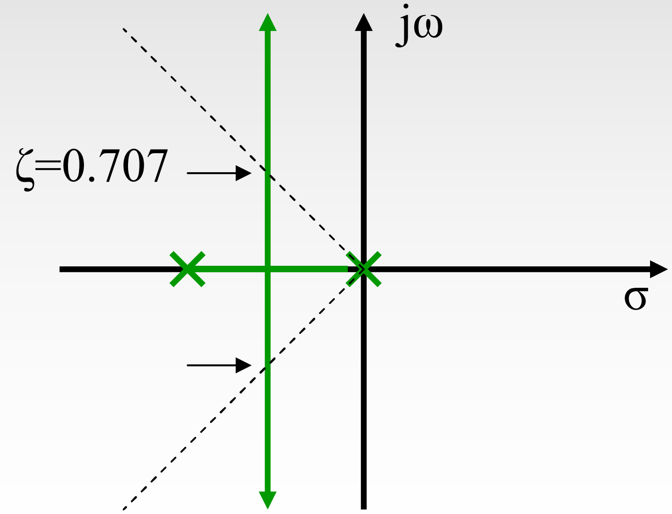
Simplified Plant

Closed-Loop Transfer Function and Root Locus



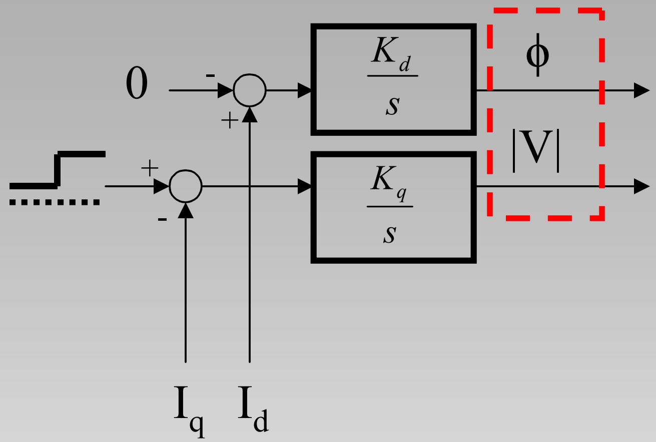
$$L(s) = G_c G_p H = \frac{K_q}{R_a s(\tau_d s + 1)}$$

$$G_{cl}(s) = \frac{G_c G_p}{1 + G_c G_p H} = \frac{K_q \tau_d s + K_q}{R_a \tau_d s^2 + R_a s + K_q}$$



Controller Simulations

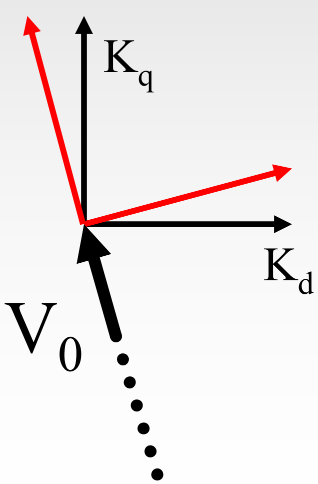
A more fair transient response comparison.



K_d = 1.0 rad/A/s (Is this fair?)

K_q = {1.2, 1.6, 2.5} V/A/s

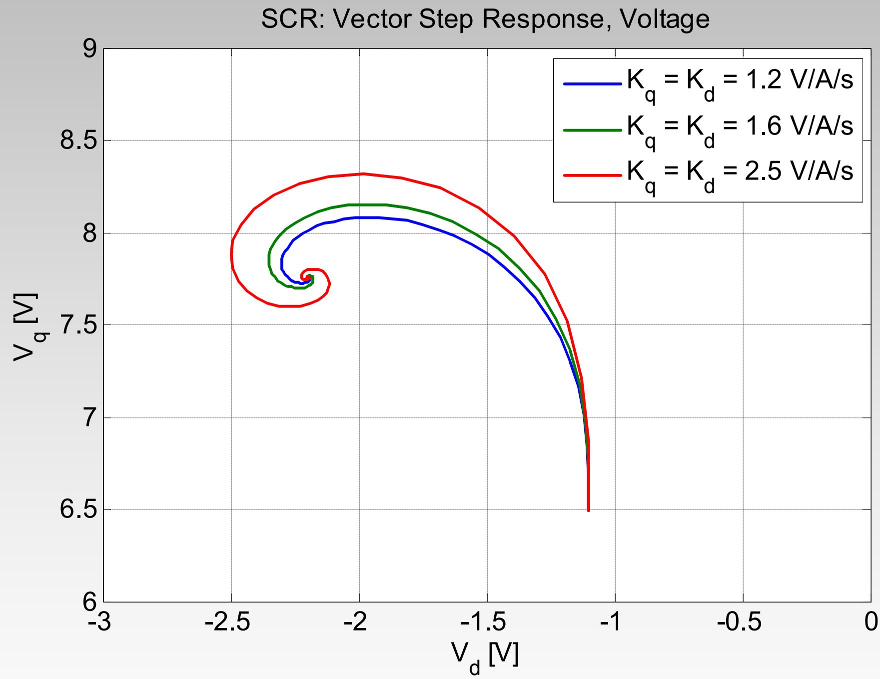
One possible way to make a more fair comparison is by using the initial voltage vector to normalize the new d-axis gain:

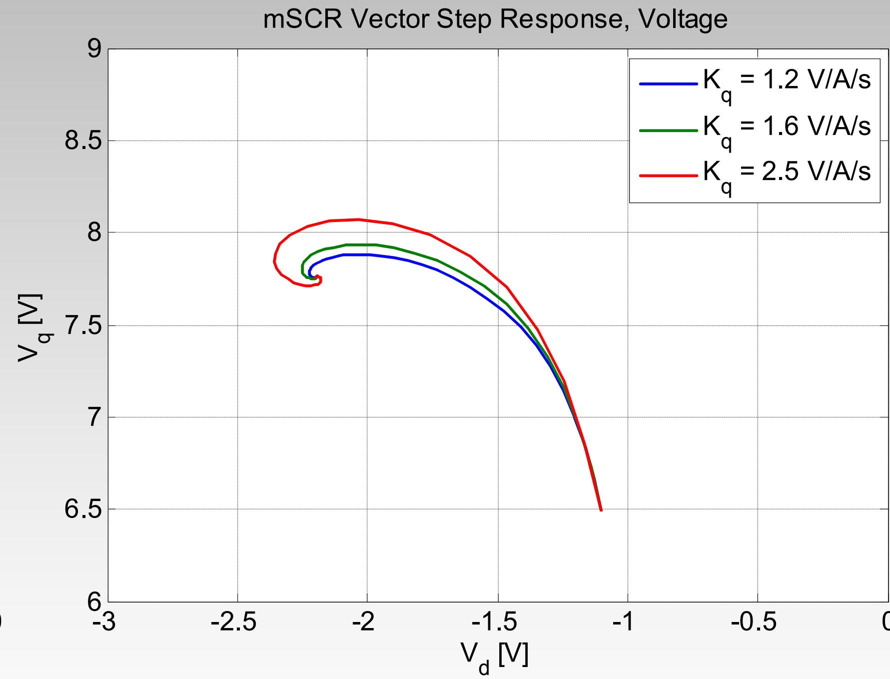


$$K_d = \frac{\{1.2 \quad 1.6 \quad 2.2\}}{|V_0|} \frac{rad}{A \cdot s}$$

Controller Simulations

A more fair transient response comparison.





Controller Simulations

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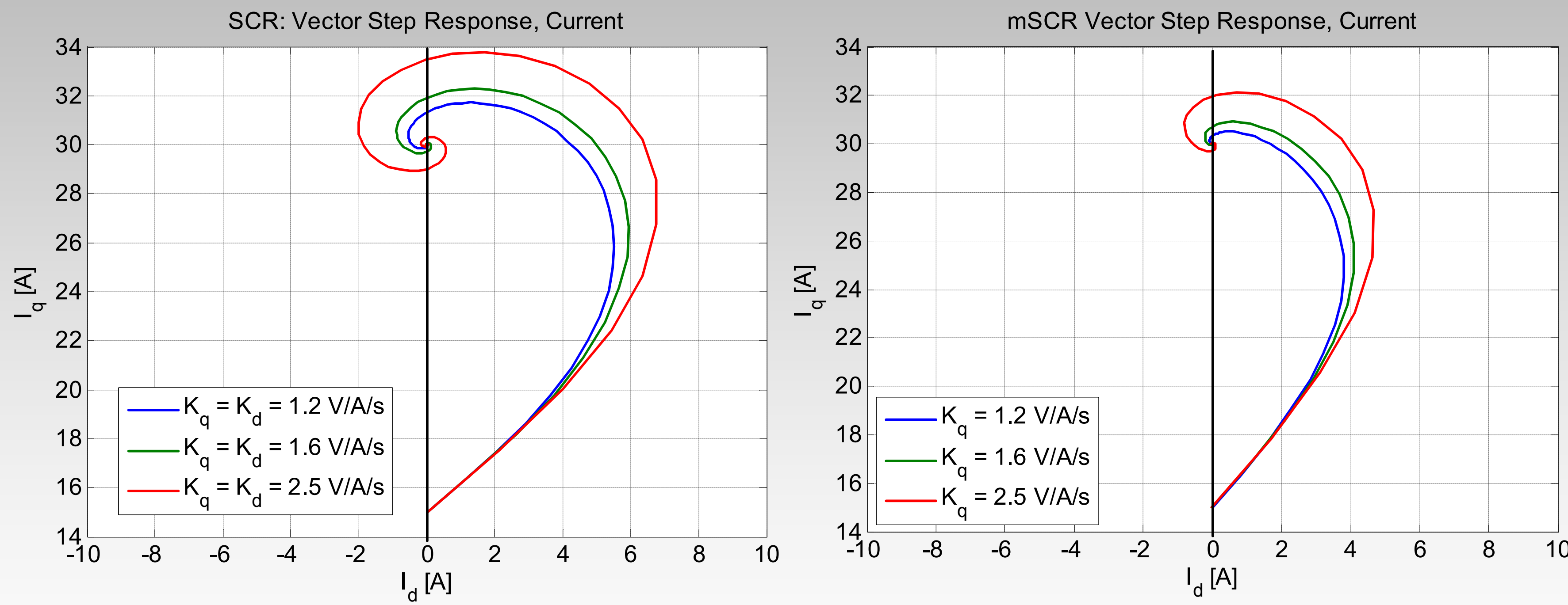
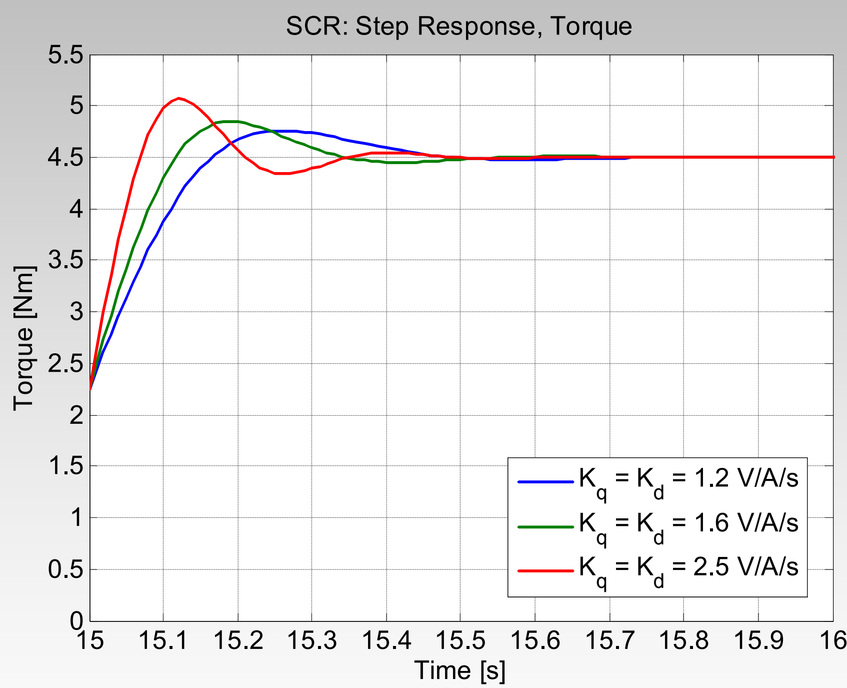
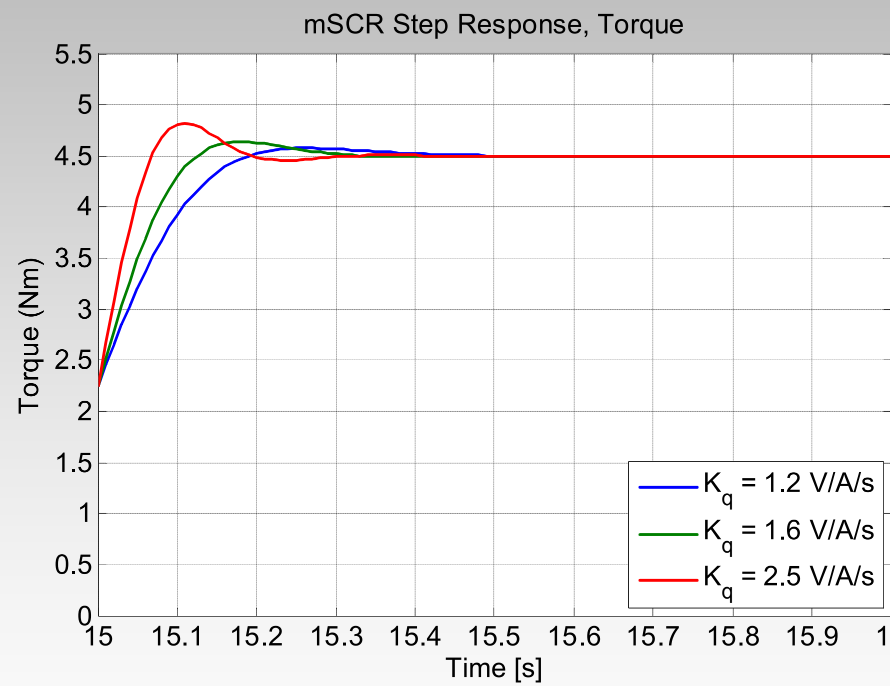


Controller Simulations

A more fair transient response comparison.





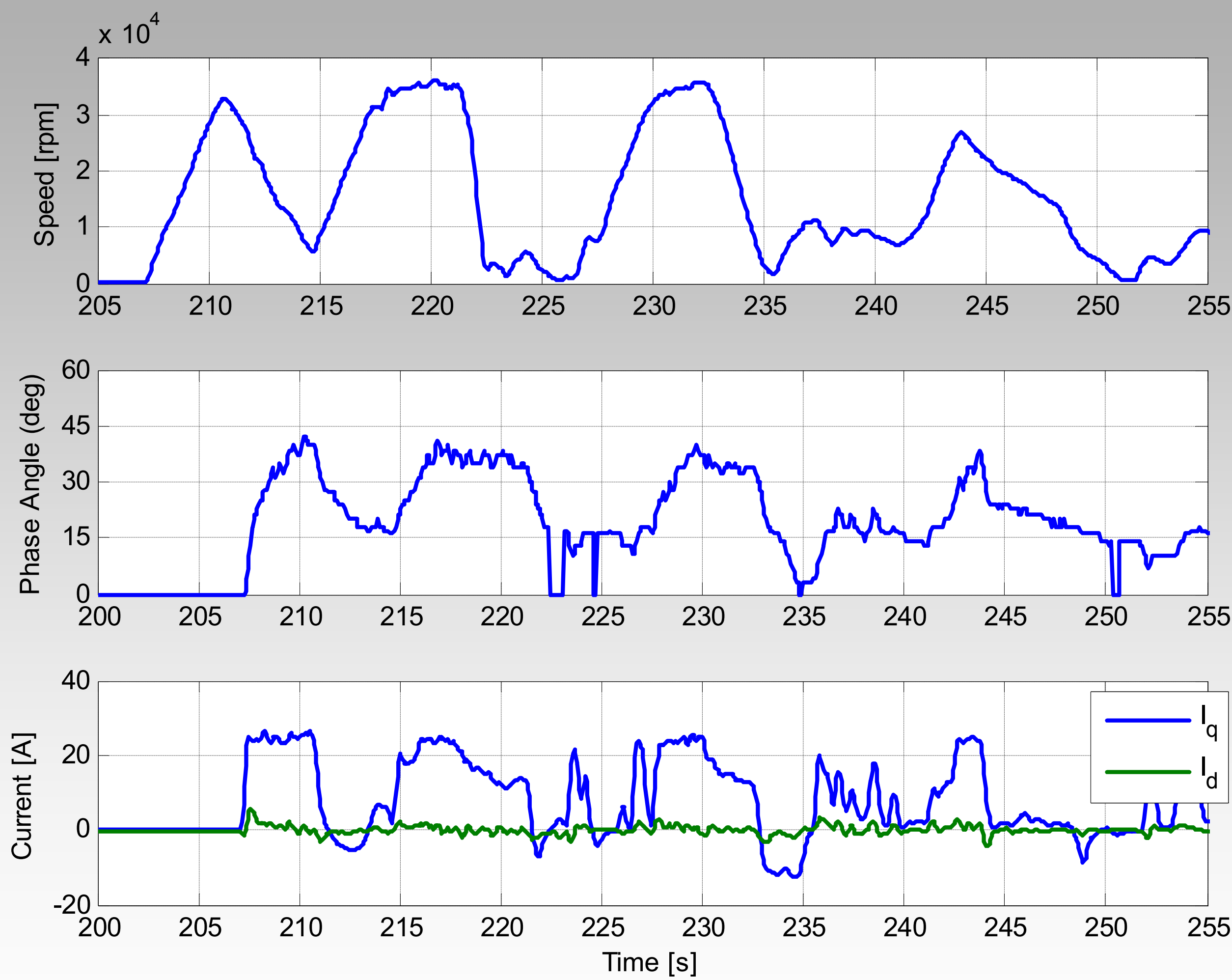
High Speed Operation

- Sensing and control becomes more difficult as speed increases:
  - $\omega L \approx R$, large phase angle.
  - Significant lag due to current sensing / AC-side filtering.
  - Analysis of digital effects (sampling, fitlering) becomes important.

- Poles: 2
- Max Speed: 35,000RPM (without field weakening)

  $\omega = 3{,}665\text{rad/s},\ f = 583\text{Hz}$
- Current sensor phase lag with components specified: ~20º!

High Speed Operation



Error Handling and Failsafes

- Hall effect sensor failure presents a significant risk to the controller.

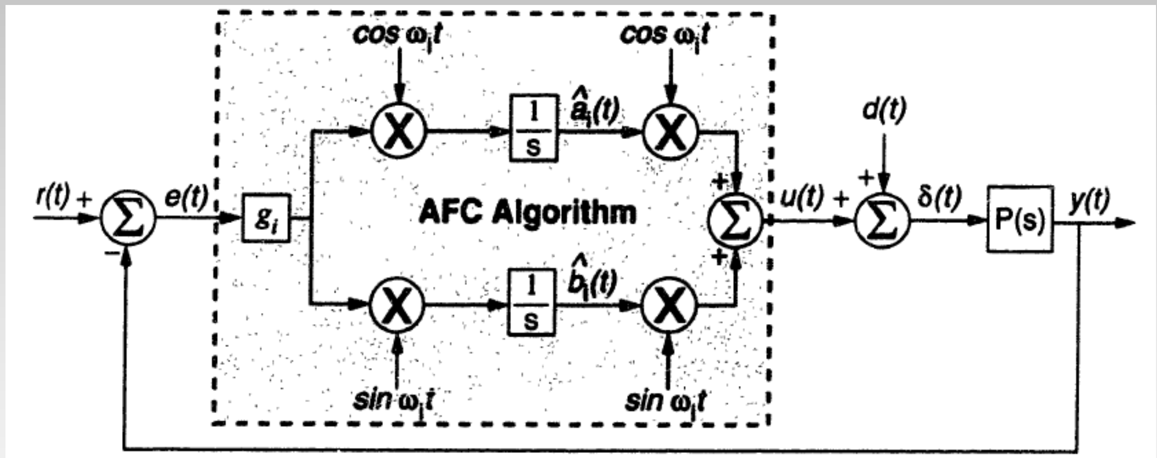
Failure Mode	Effect	Countermeasure
The entire sensor cable becomes unplugged.	Comlete loss of ability to commutate the motor.	Pull-down resistors take the sensor state to {0,0,0}, which is invalid. The output driver shuts down. Motor coasts.
Transient sensor glitch. < 1/6 cycle (single sensor glitch)	An unexpected state transition, resulting in large current/torque transient when voltage vector is applied at the wrong angle.	If new state is not as expected, trust rotor speed interpolation for the next 60º segment.
Permanent sensor failure. > 1/6 cycle	Repeated loss of two states per cycle.	Follow same rules as above, but with a counter that talleys unexpected state transitions per unit time. If larger than some threshold, shut down.

. . .

- Sensorless or hybrid techniques will significantly change the FMEA.
- Future work: Ability to switch to sensorless control if a Hall effect sensor fault is detected.

Connection to Adaptive Feed-Forward Cancellation

- The SCR and mSCR are applications of adaptive feed-forward cancellation (AFC) to three-phase variables.
- In one implementation of AFC, a feed-forward path allows for zero-error tracking of a sinusoidal input at a specific frequency:



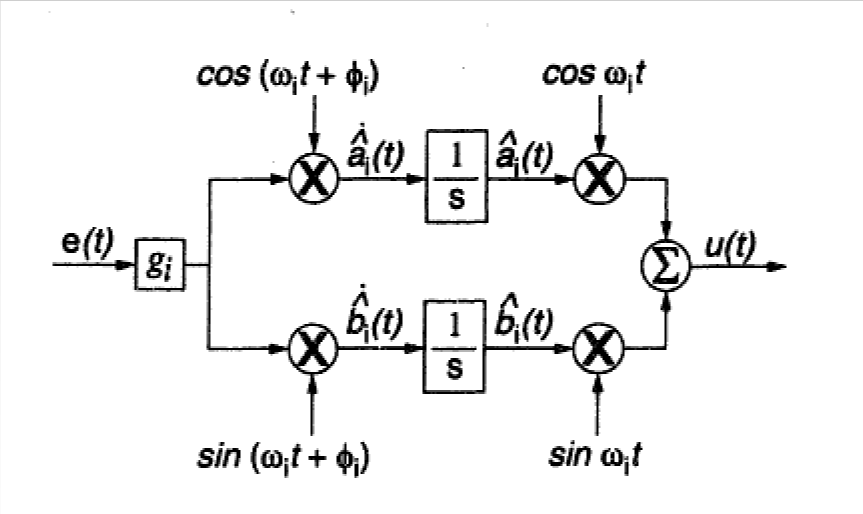
Adaptive Feedforward Cancellation Closed-Loop Block Diagram

Reference:
Cattell, Joseph H. *Adaptive Feedforward Cancellation Viewied from an Oscillator Amplitude Control Perspective.* S.M. Thesis, Massachusetts Institute of Technology, 2003.

Connection to Adaptive Feed-Forward Cancellation

- By manipulating the block diagram of a the SCR, focusing on the amplitude of a single phase of current, the SCR can be related to single-oscillator AFC (not proven here).

- The modified SCR is related to single-oscillator AFC *with* a phase advance offset, which has been proven to improve transient response.



- In both cases, the Park Transform provides the sinuosoidal multiplier for the input and output.

- In AFC with phase advance, ϕ_i is set as the plant phase angle (initial voltage vector angle).

Reference:
Cattell, Joseph H. *Adaptive Feedforward Cancellation Viewied from an Oscillator Amplitude Control Perspective.* S.M. Thesis, Massachusetts Institute of Technology, 2003.